

Crossing Numbers of Beyond-Planar Graphs Revisited

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Crossing number $cr(G)$ of a graph G

Minimum number of crossings across all possible drawings of G .

Beyond-planar graph family F

Family of graphs that admit a drawing which does not contain the forbidden configuration that describes F .

Restricted crossing number $cr_F(G)$ of a graph G

Minimum number of crossings over all drawings of G restricted by F .

Crossing ratio ρ_F of a family F

Largest ratio $cr_F(G)/cr(G)$ over all graphs G in F .

Previous work

Chimani et al. [1] gave bounds for the 1-planar, k -quasi-planar and fan-planar crossing ratio.

[1] Crossing numbers of beyond-planar graphs.

M. Chimani, P. Kindermann, F. Montecchiani, P. Valtr

Proc. 27th International Symposium on Graph Drawing (2019)

Contribution

We extend the results by Chimani et al. [1] to the main classes of beyond-planar graphs.

All bounds on the crossing ratios are summarized on the right.

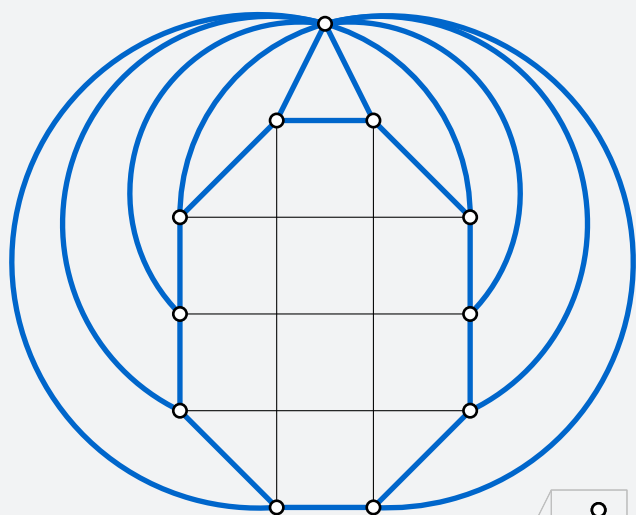
Bounds that are tight for a fixed k are indicated in blue.

Full version:

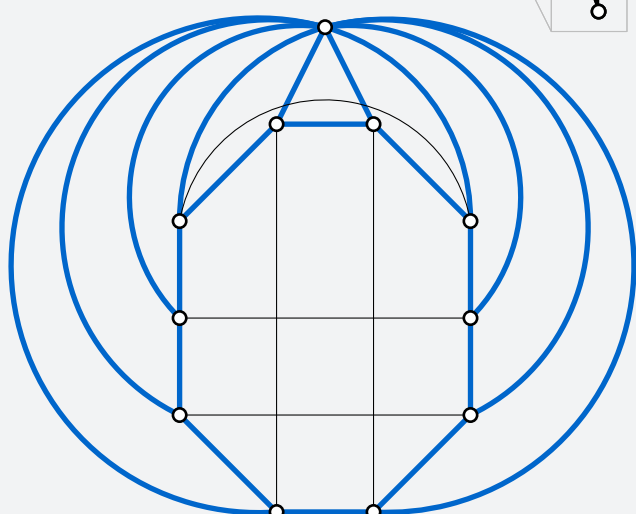
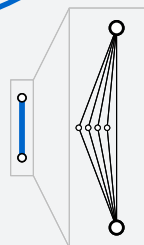


Family	Forbidden Configurations	Lowerbound	Upperbound
k -planar	An edge crossed more than k times	$\Omega(n/k)$	$O(k\sqrt{kn})$
k -quasi-planar	k pairwise crossing edges	$\Omega(n/k^3)$ [1]	$f(k)n^2 \log^2 n$ [1]
Fan-planar	Two independent edges crossing a third or two adjacent edges crossing another edge from different "sides"	$\Omega(n)$ [1]	$O(n^2)$ [1]
(k, l) -grid-free	Set of k edges such that each edge crosses each edge from a set of l edges.	$\Omega\left(\frac{n}{kl(k+l)}\right)$	$g(k, l)n^2$
k -gap-planar	More than k crossings mapped to an edge in an optimal mapping	$\Omega(n/k^3)$	$O(k\sqrt{kn})$
Skewness- k	Set of crossings not covered by at most k edges	$\Omega(n/k)$	$O(kn + k^2)$
k -apex	Set of crossings not covered by at most k vertices	$\Omega(n/k)$	$O(k^2n^2 + k^4)$
Planarly connected	Two crossing edges that do not have two of their endpoint connected by a crossing-free edge	$\Omega(n^2)$	$O(n^2)$
k -fan-crossing-free	An edge that crosses k adjacent edges	$\Omega(n^2/k^3)$	$O(k^2n^2)$
Straight-line RAC	Two edges crossing at an angle $< \frac{\pi}{2}$	$\Omega(n^2)$	$O(n^2)$

2,3-grid-free

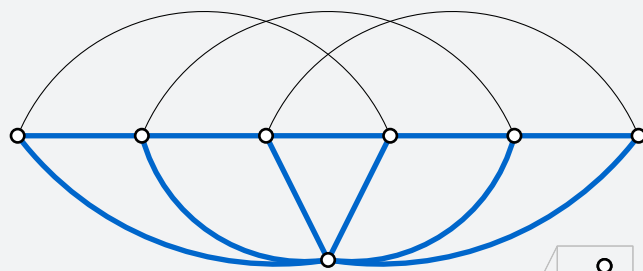


$cr(G) \leq 6$

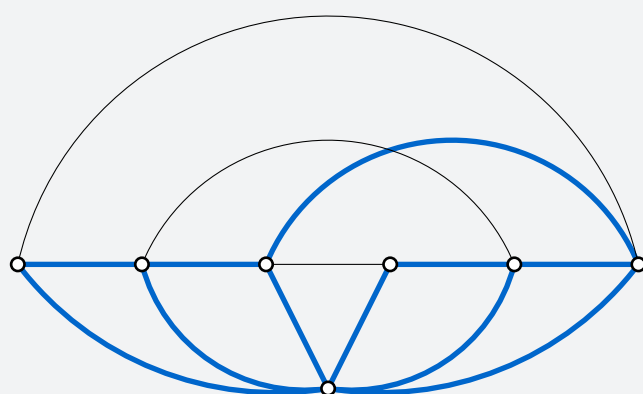
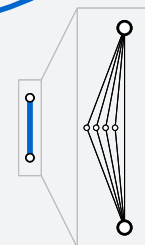


$\rho_{k,l\text{-grid-free}} \in \Omega(n)$

Skewness-1

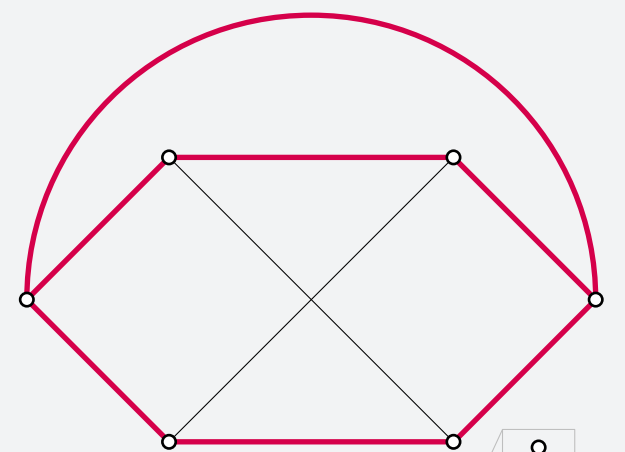


$cr(G) \leq 3$

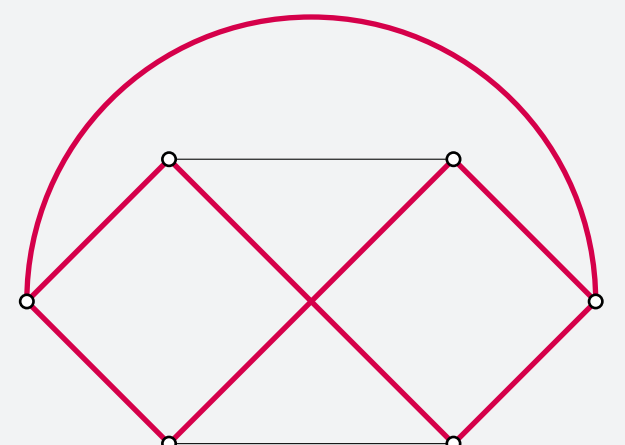
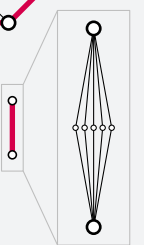


$\rho_{\text{skew-1}} \in \Omega(n)$

Planarly connected



$cr(G) \leq 1$



$\rho_{\text{pl-con}} \in \Omega(n^2)$