Compact Convex Planar Grid Drawings of Graphs with Constant Edge-Vertex Resolution

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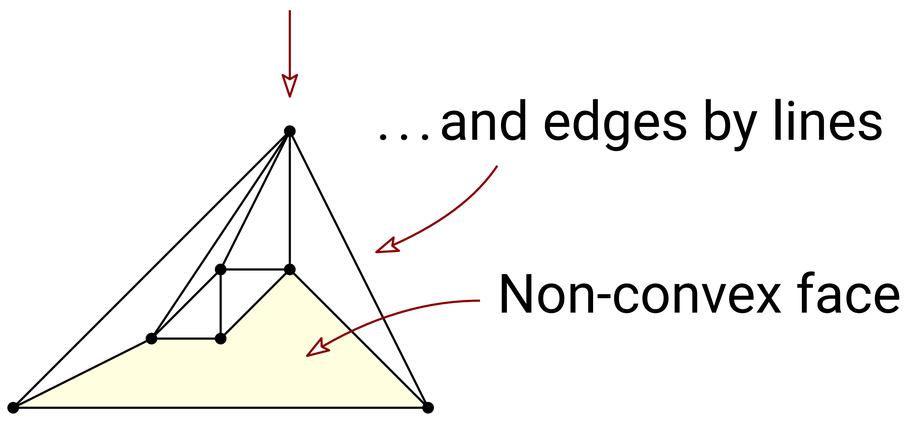
Graph Drawing in Theory

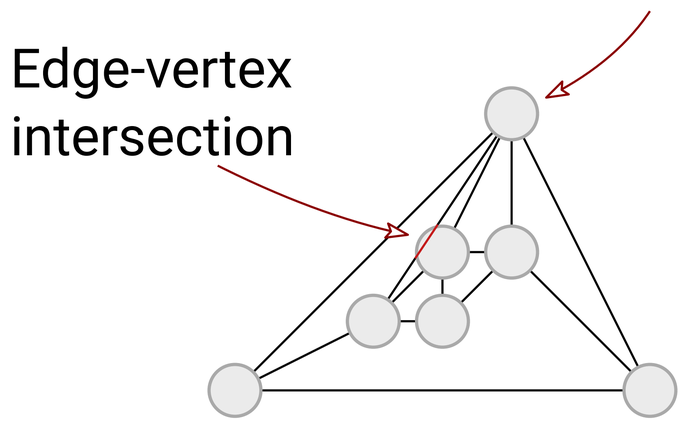
Vertices represented by points



Planar straight-line drawing

. . . and in Practice

Vertices drawn as (unit) disks



Goal: Small area bounds avoiding edge-vertex intersections

Previous Work

Claim (Chrobak et al. 1995)
Every 3-connected planar graph with n vertices admits a convex planar straight-line drawing on a grid of size $(3n-7)\times(3n-7)/2$ with edge-vertex resolution at least $\frac{1}{2}$.

Recently proved for maximal planar graphs (Bekos et al. 2021).

Convexity trivially implied

Contribution

Theorem
Every 3-connected plane graph with n vertices and f faces admits a convex planar straight-line drawing with edge-vertex resolution at least $\frac{1}{2}$ on a grid of size $(n-2+a)\times(n-2+a)$, where $a=\min\{f, n-3\}$.

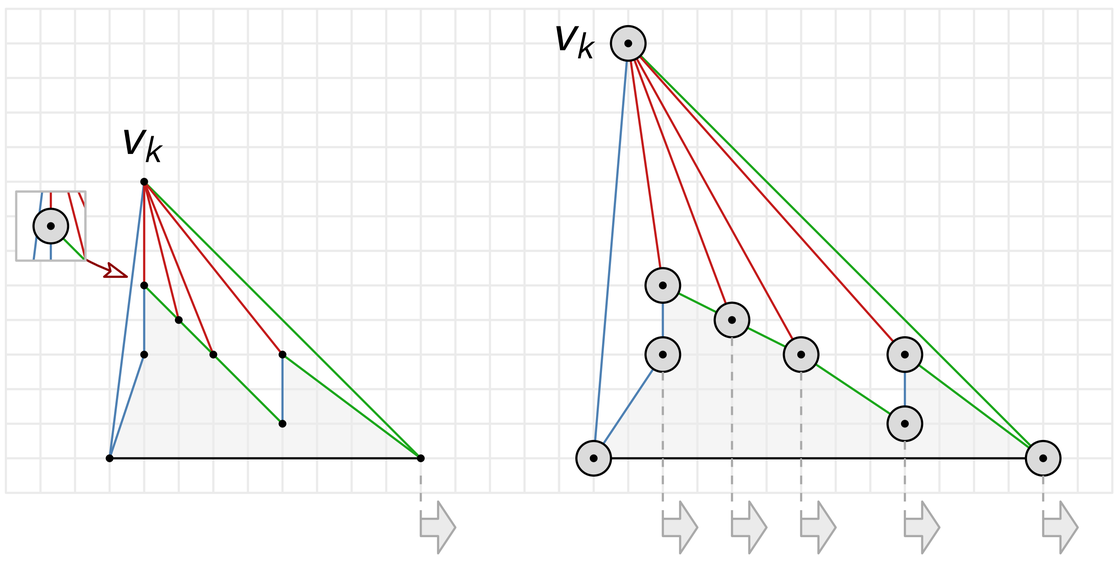
- Maximal planar $\Rightarrow (2n-5)\times(2n-5)$
- 3-connected cubic planar $\Rightarrow 3n/2\times 3n/2$

Note: Improves also upon the claim

Algorithm

Main idea

- Utilizes canonical ordering for 3-connected planar graphs [4]
- Follows the lines of Chrobak & Kant [3]
- Chains $\rightarrow$ as in the original algorithm
- Singletons $\rightarrow$ require extra shifts
- Area analysis based on a charging scheme using faces and vertices



Original algorithm Modified approach

[1] M. Bekos, M. Gronemann, F. Montecchiani, D. Pálvölgyi, A. Symvonis, L. Theocharous: Grid drawings of graphs with constant edge-vertex resolution. Comp. Geom. 98: 101789 (2021)

[2] M. Chrobak, M. T. Goodrich, R. Tamassia: Convex Drawings of Graphs in Two and Three Dimensions (Preliminary Version). SCG 1996: 319-328

[3] M. Chrobak, G. Kant: Convex Grid Drawings of 3-Connected Planar Graphs. Int. J. Comput. Geom. Appl. 7(3): 211-223 (1997)

[4] G. Kant: Drawing Planar Graphs Using the Canonical Ordering. Algorithmica 16(1): 4-32 (1996)

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