ONE-WAY k-CROSSABLE GRAPHS

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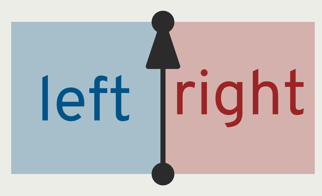
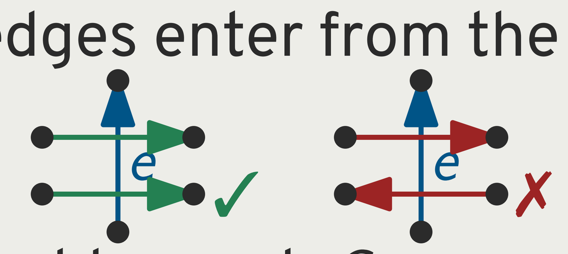
Definition: *one-way k-crossable graphs*

directed graph $\Rightarrow$ left and right side of edges well-defined



edge e is one-way crossed
$\Rightarrow$ all crossing edges enter from the same side.



one-way k-crossable graph G:
$\Rightarrow$ all edges of G are one way crossed
$\Rightarrow$ each edge is involved in at most k crossings

Our Results

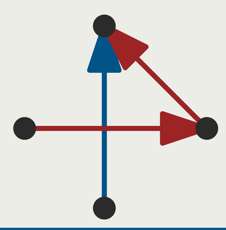
k	Density Upper Bound	Density Lower Bound
1	$4(n-2)$	$4(n-2)$
2	$\frac{13}{3}(n-2)$	$\frac{13}{3}(n-2)$
3	$5(n-2)$	$5(n-2)-2$
≥ 7	$6(n-2)$	$6(n-2)-6$

Theorem 1: *Let G be one-way k-crossable. Then, G is a bi-planar graph, i.e., its thickness is 2.*

every edge of G is assigned a color

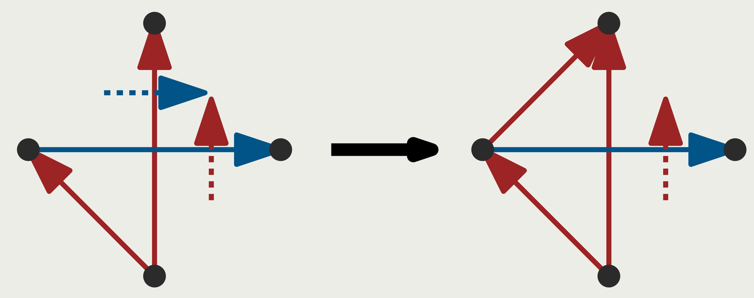
blue: edges crossed by edges coming from the left
red: remaining edges

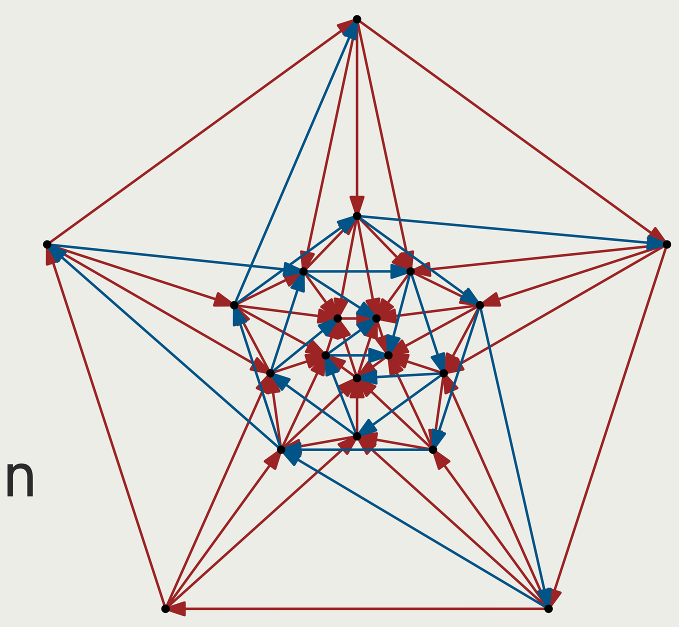
$\Rightarrow$ red and blue planar subgraphs.



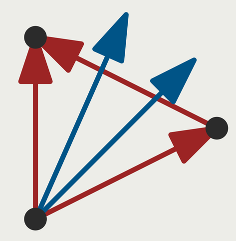
Theorem 3: *Let G be one-way 2-crossable. Then, G has at most $\frac{13}{3}(n-2)$ edges which is a tight bound.*

W.l.o.g. every blue edge has its endvertices in red triangles

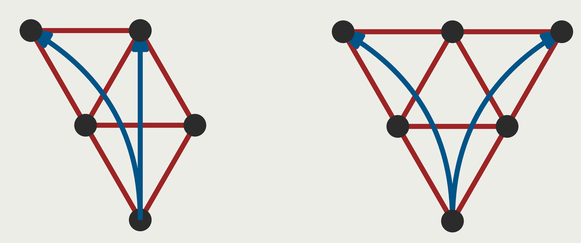


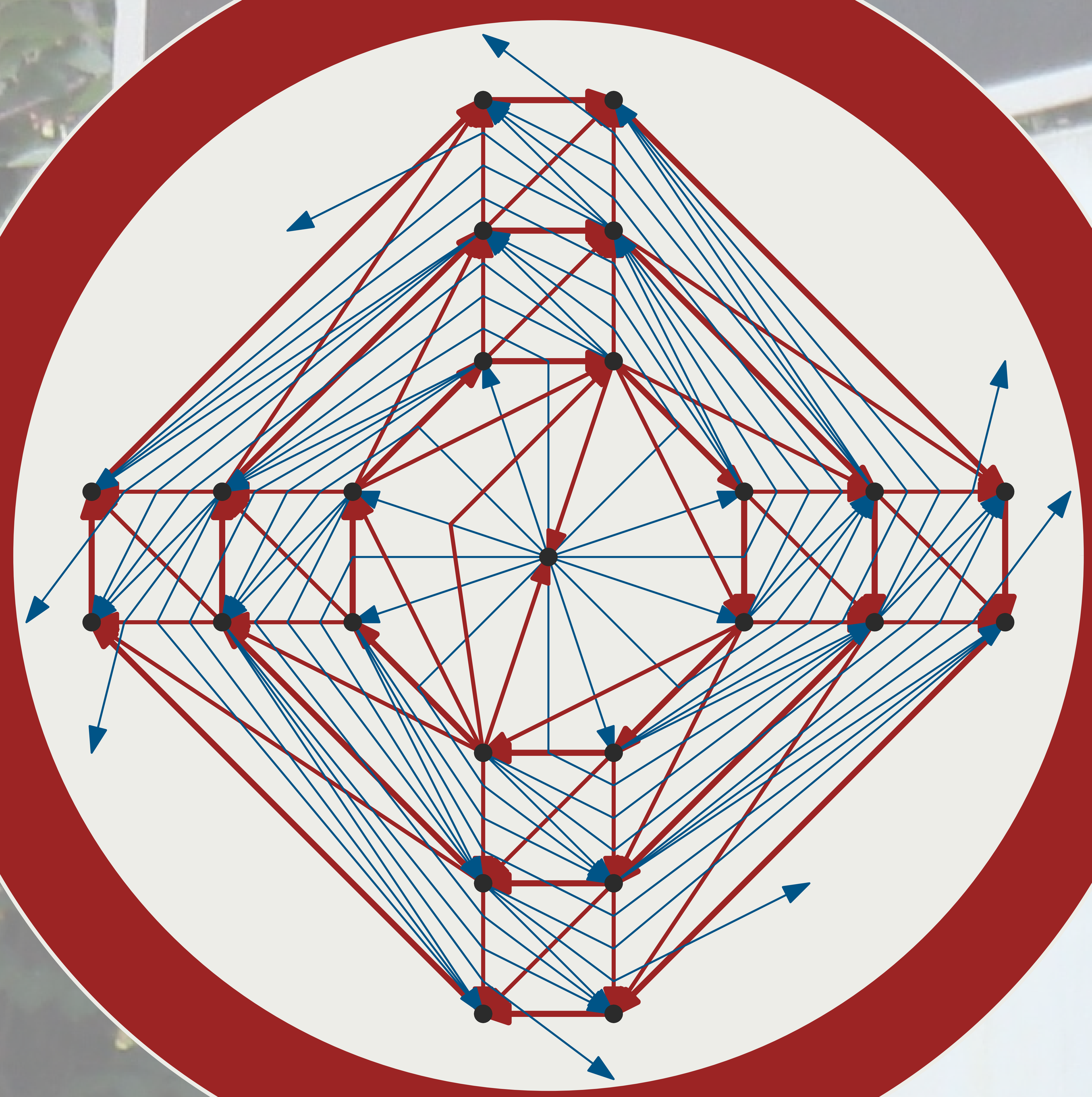


Each red triangle contains the common endvertex of ≤ 2 blue edges.



3 red triangles contain endpoints of $\leq$ 2 blue edges.

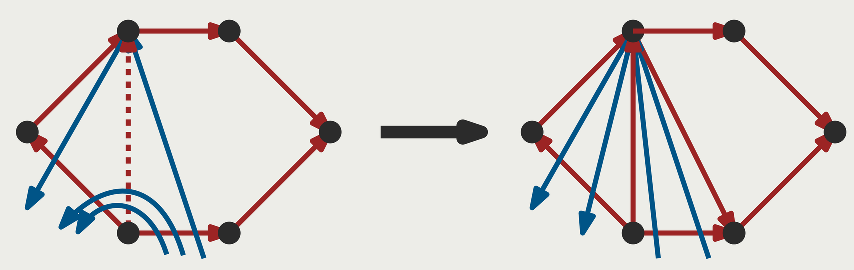




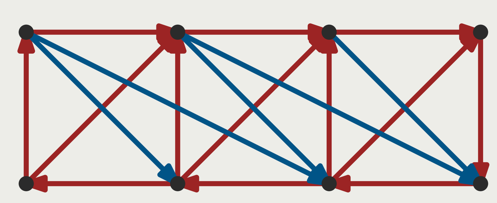
Theorem 2: *There are infinitely many n-vertex one-way 7-crossable graphs with m=6n-18 edges.*

Theorem 4: *G one-way 3-crossable $\Rightarrow$ G has at most $5(n-2)$ edges.*

W.l.o.g. every blue edge has its endvertices in red triangles



3 adjacent blue edges account for 4 red triangles or each red triangle contains $\leq$ 2 adjacent blue edges.



Open Problems:

Is there a $k < 7$ that gives a density of $6n - O(1)$?

Recognition of one-way k-crossable graphs

References:

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[2] Bekos, M.A., Kaufmann, M., Raftopoulou, C.N.: On optimal 2- and 3-planar graphs. CoRR (2017)
[3] Didimo, W., Liotta, G., Montecchiani, F.: A survey on graph drawing beyond planarity. ACM Comput. Surv. 52(1), 4:1–4:37 (2019)
[4] Pach, J., Tóth, G.: Graphs drawn with few crossings per edge. Comb. 17(3), 427–439 (1997)