Unit Disk Representations

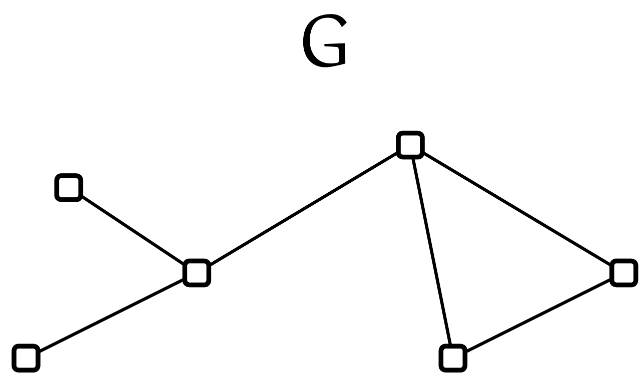
of Embedded Trees, Outerplanar and Multi-Legged Graphs

Sujoy Bhore, Maarten Löffler, Soeren Nickel and Martin Nöllenburg

16.09.2021 · GD 2021

Unit Disk Graphs

A graph G is a **unit disk graph**



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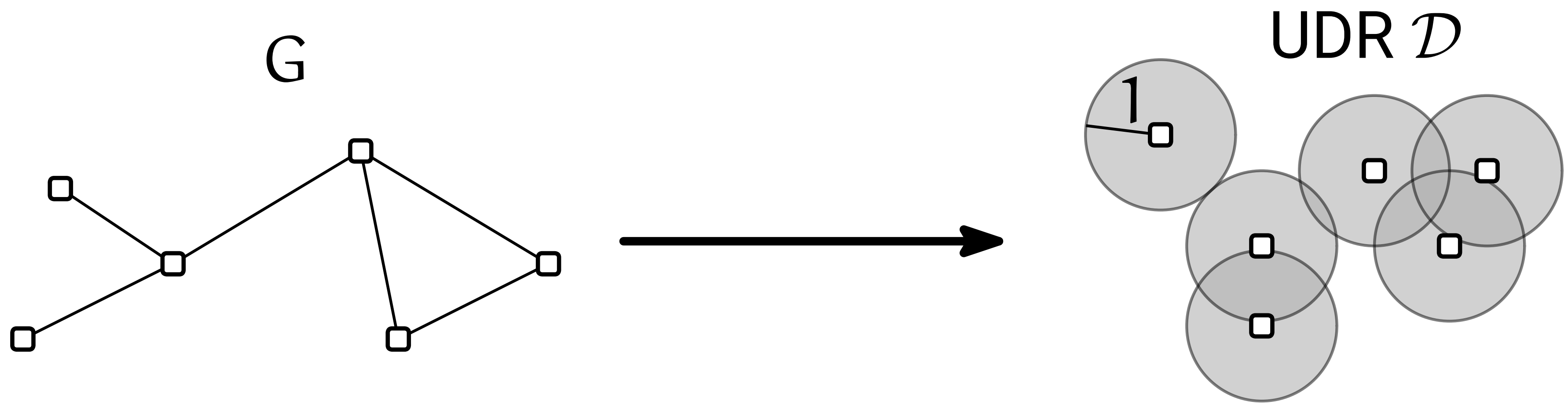
A graph G is a **unit disk graph** if it has a **unit disk representation** $\mathcal{D}$.



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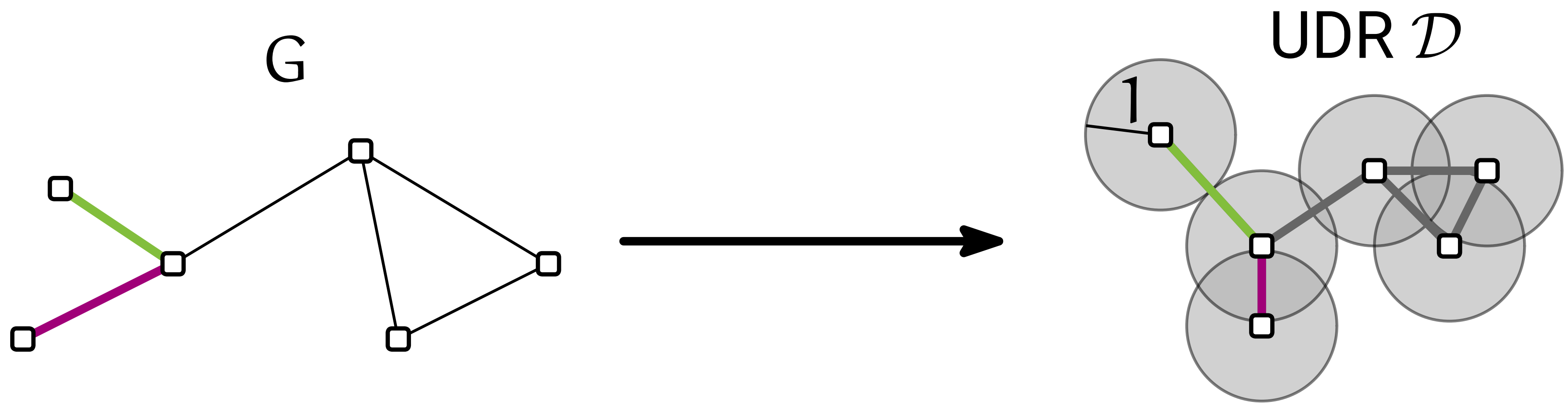

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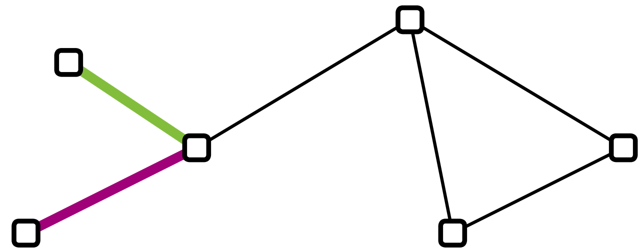
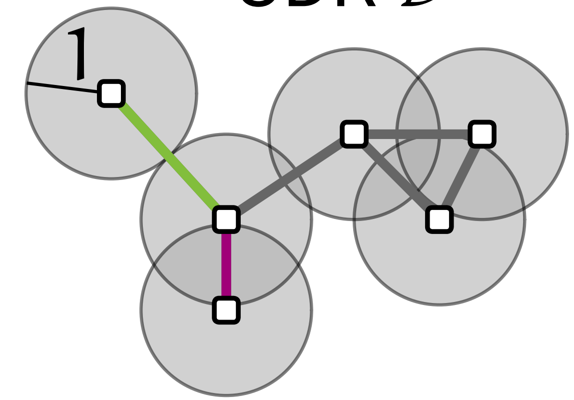
G



UDR $\mathcal{D}$



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- If all disks are **interior-disjoint**, then $\mathcal{D}$ is a unit disk **contact** representation.

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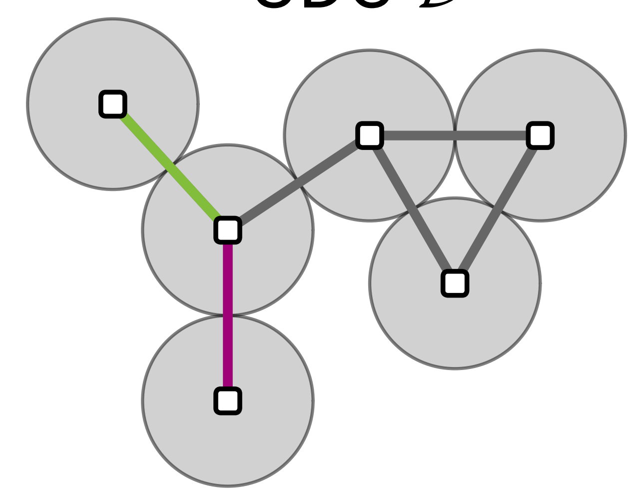

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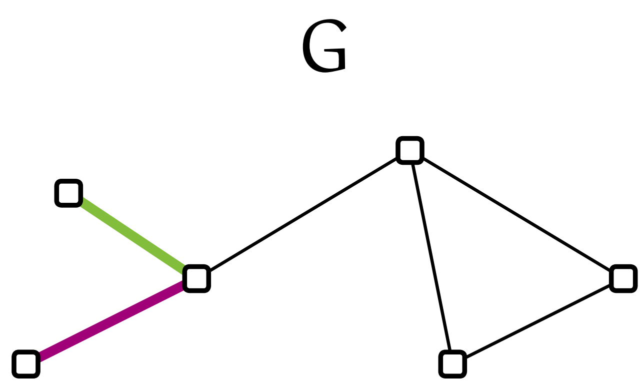

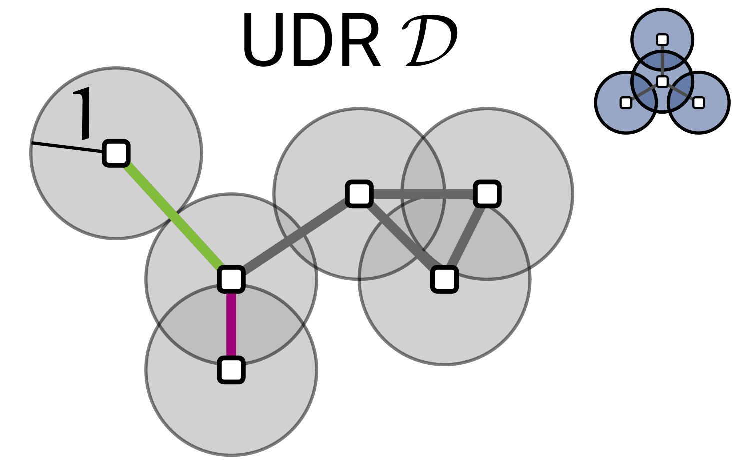

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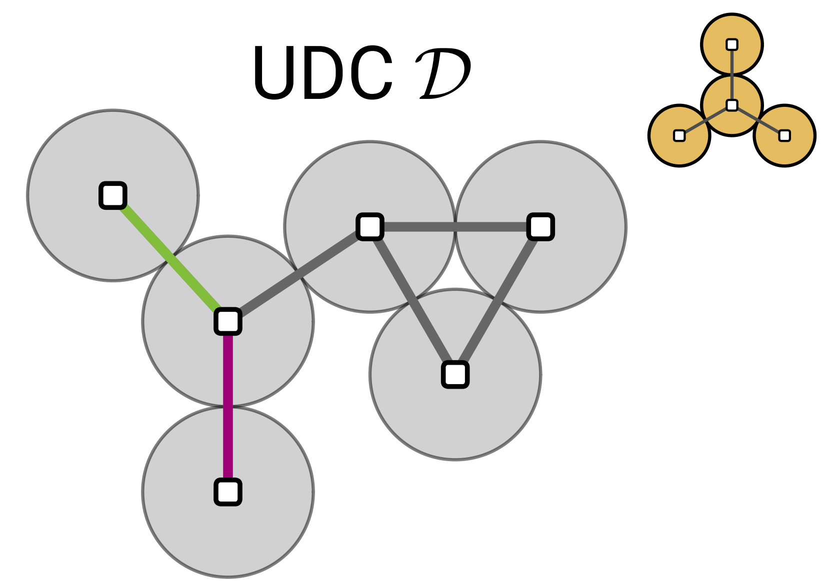

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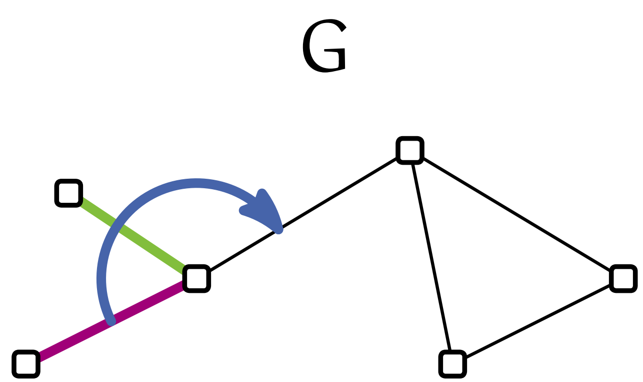

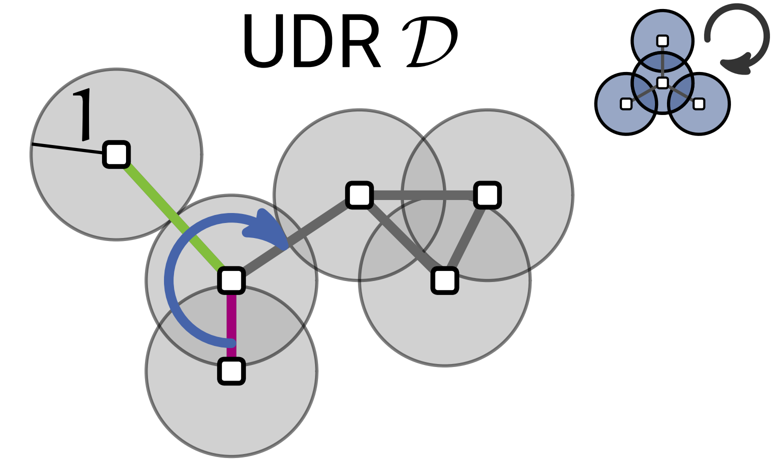

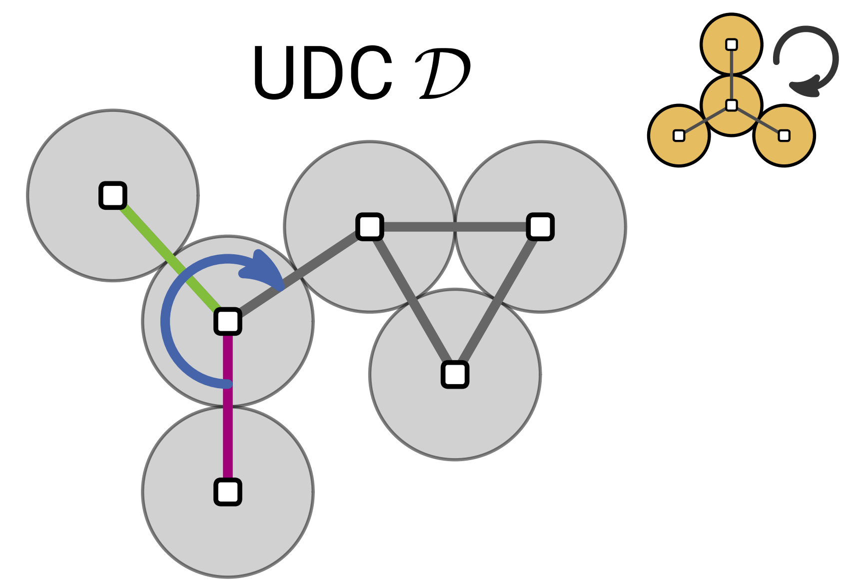

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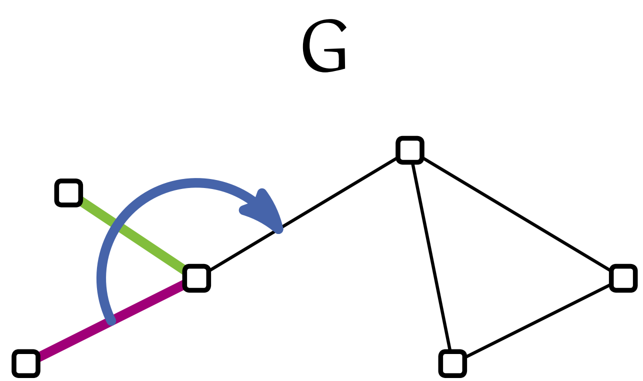

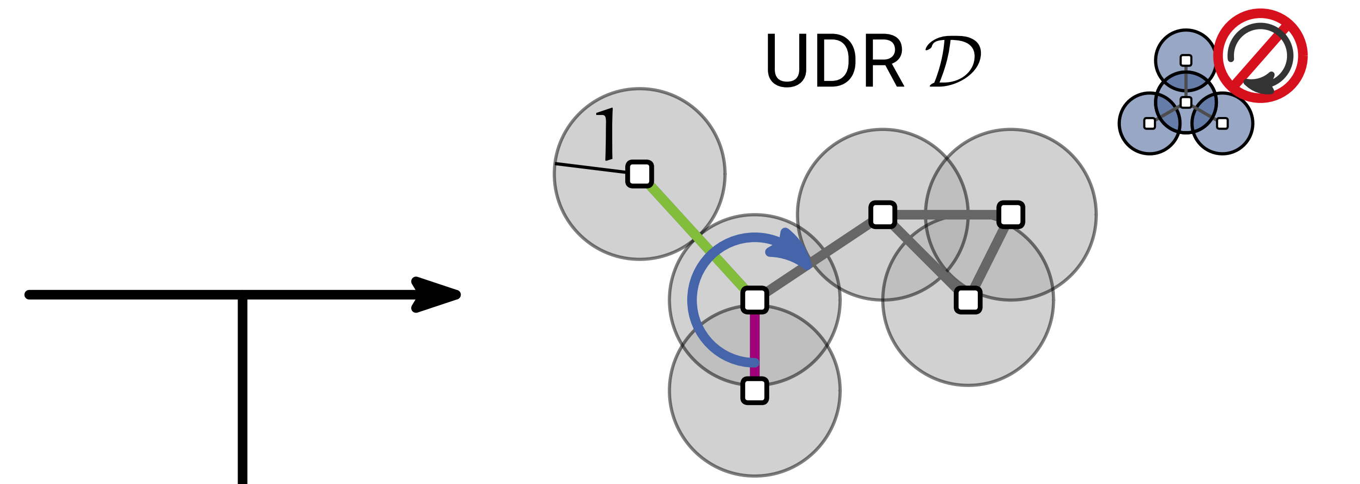

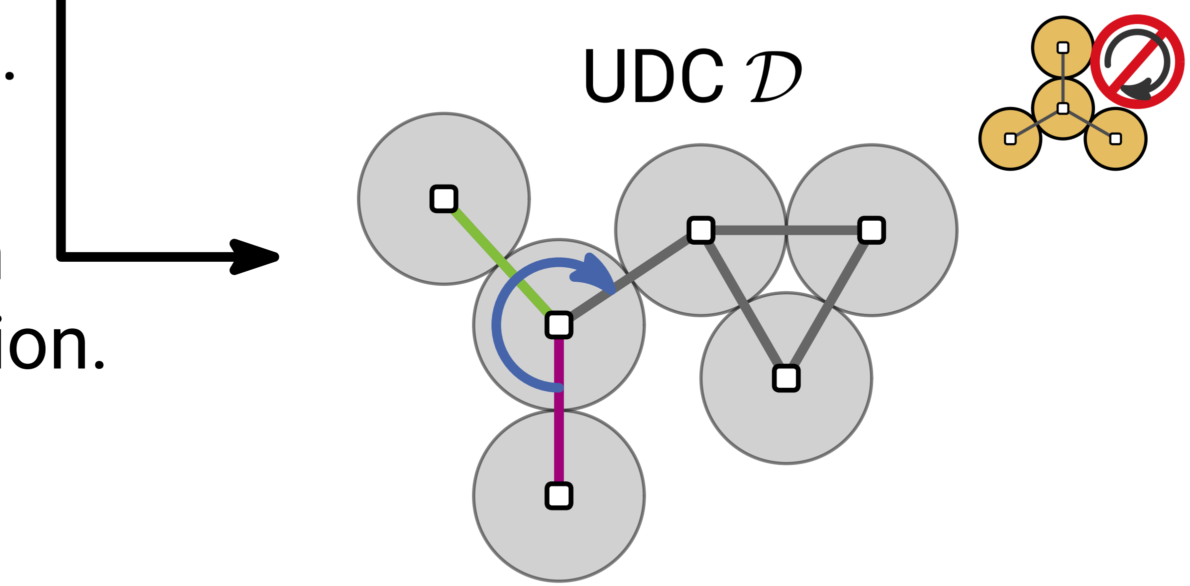

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Recognition Problem

Recognition of unit disk (contact) graphs is NP-hard for planar graphs.[1]

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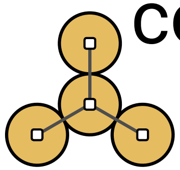 contact graphs	NP-hard[1]			
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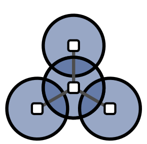 disk graphs	NP-hard[1]			
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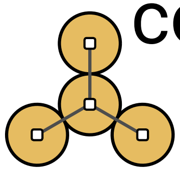 contact graphs	NP-hard[1]	NP-hard[2]		$\mathcal{O}(n)$ [2]
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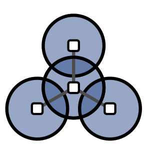 disk graphs	NP-hard[1]			
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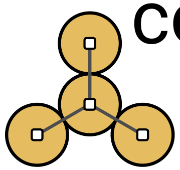

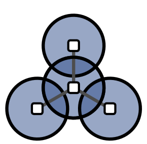

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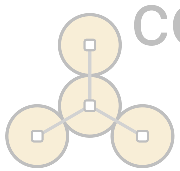

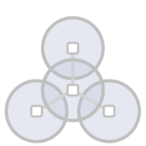

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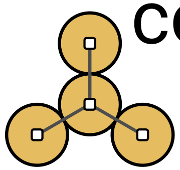 contact graphs	NP-hard[1]	NP-hard[2]	?	$\mathcal{O}(n)$ [2]
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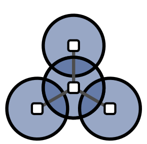 disk graphs	NP-hard[1]	NP-hard	?	$\mathcal{O}(n)$
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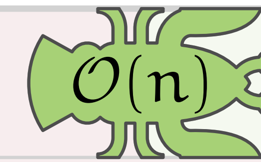
weak contact	NP-hard[4]			 $\mathcal{O}(n)$ [4]
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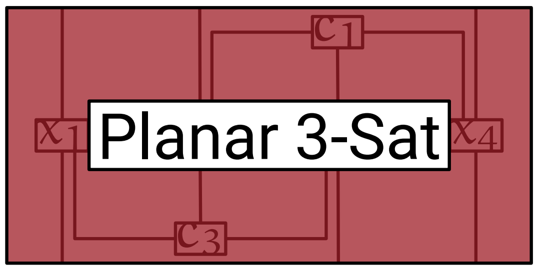

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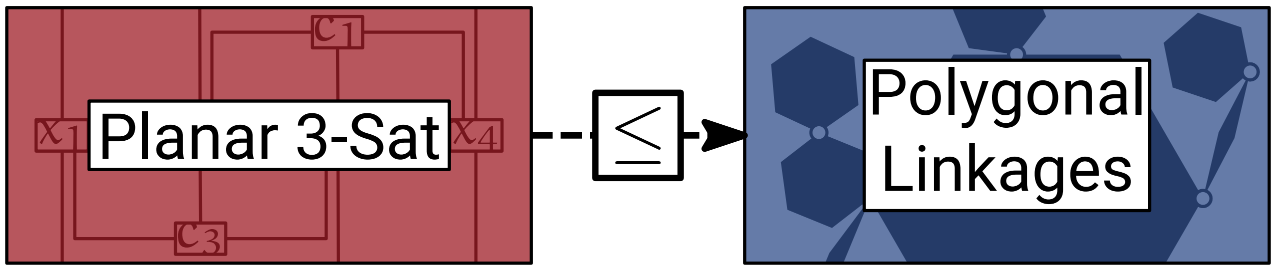

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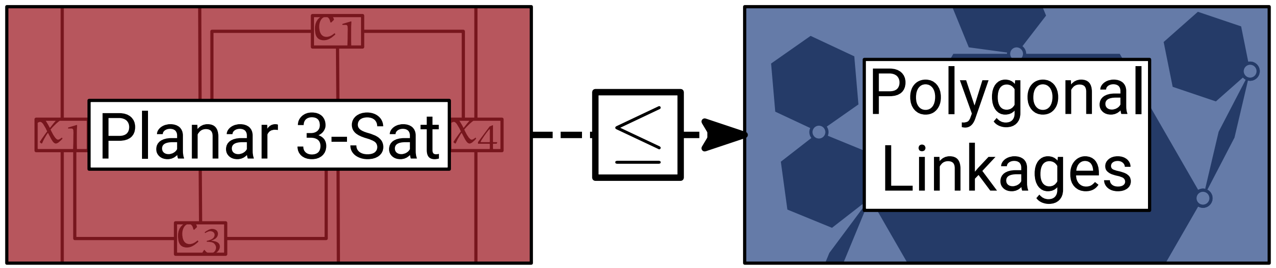

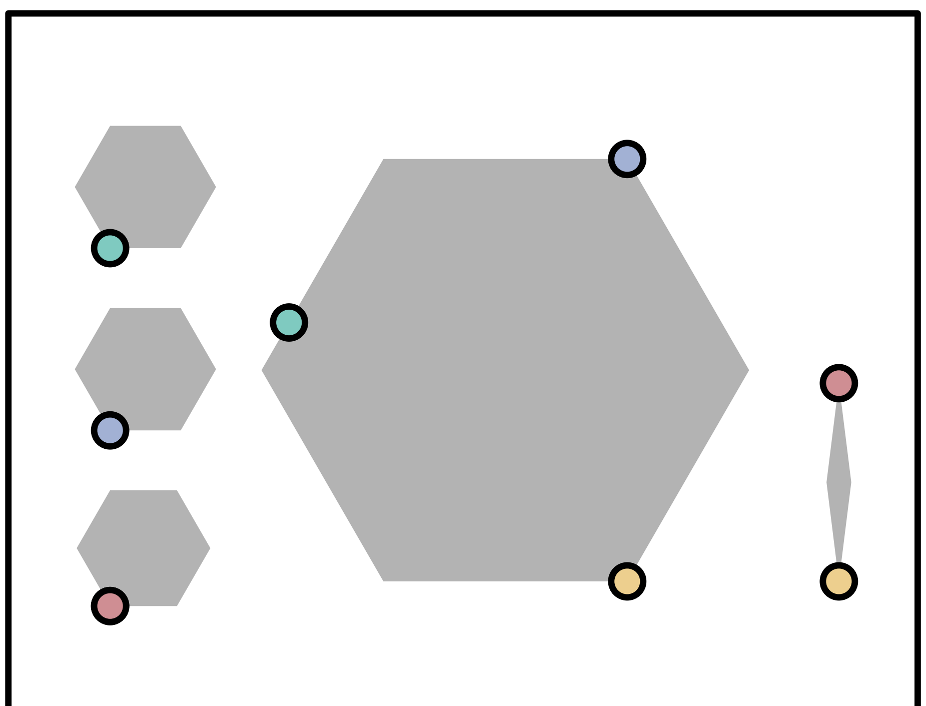

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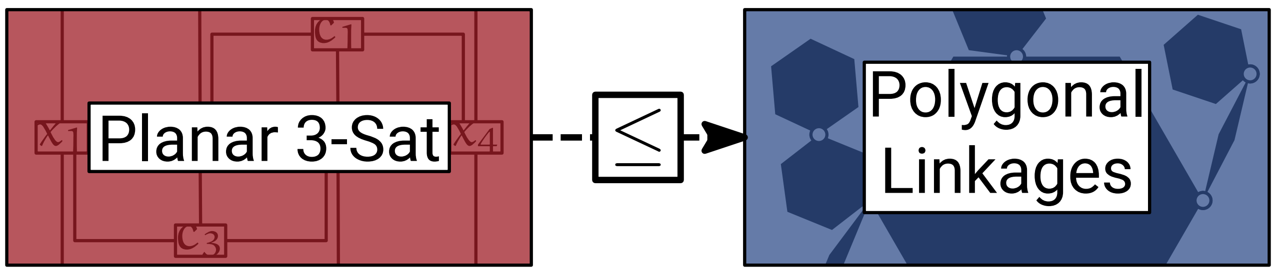

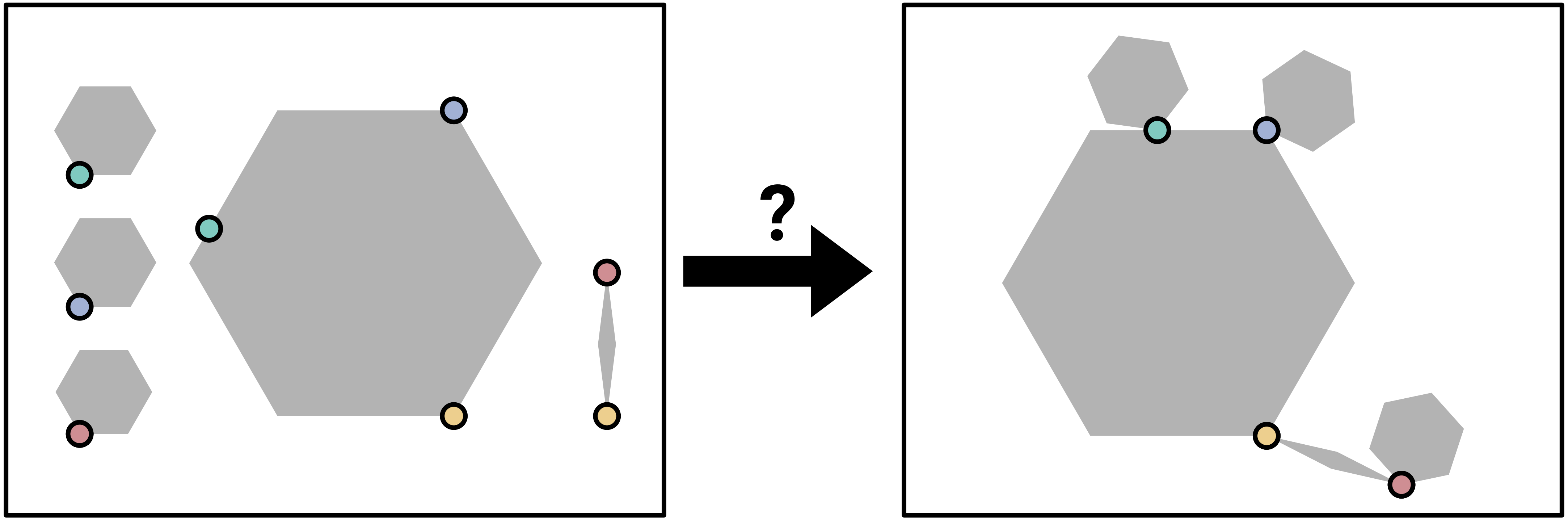

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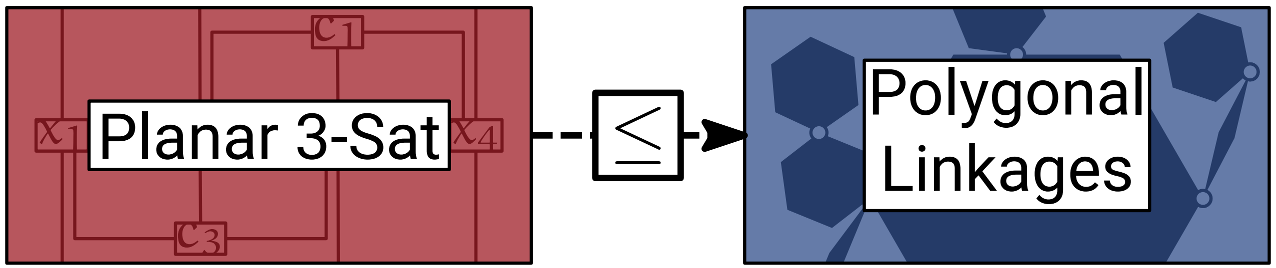

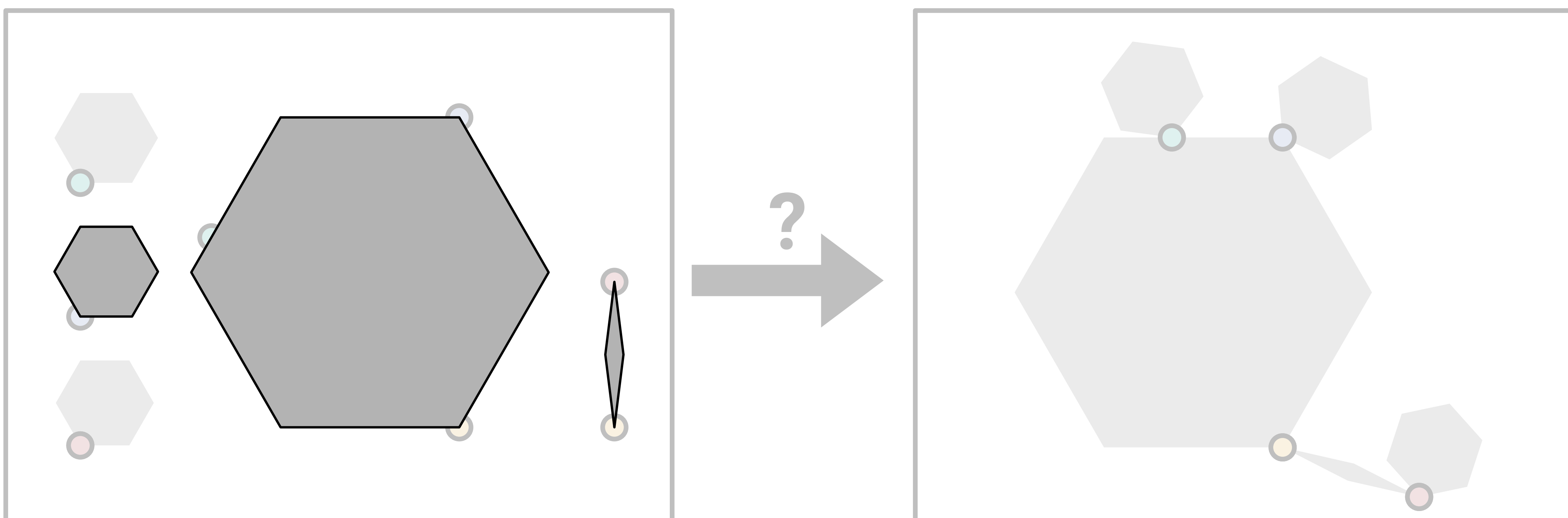

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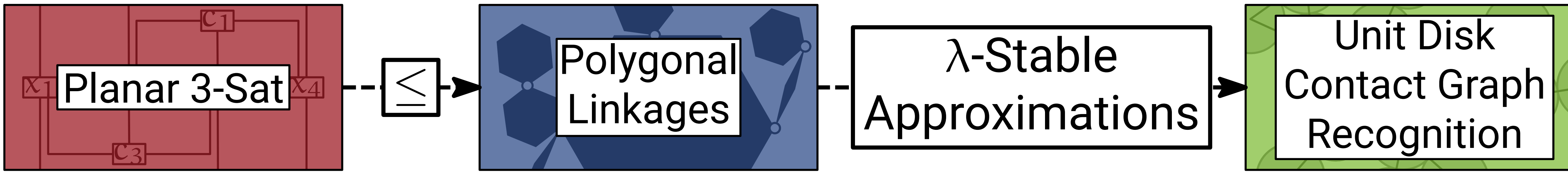

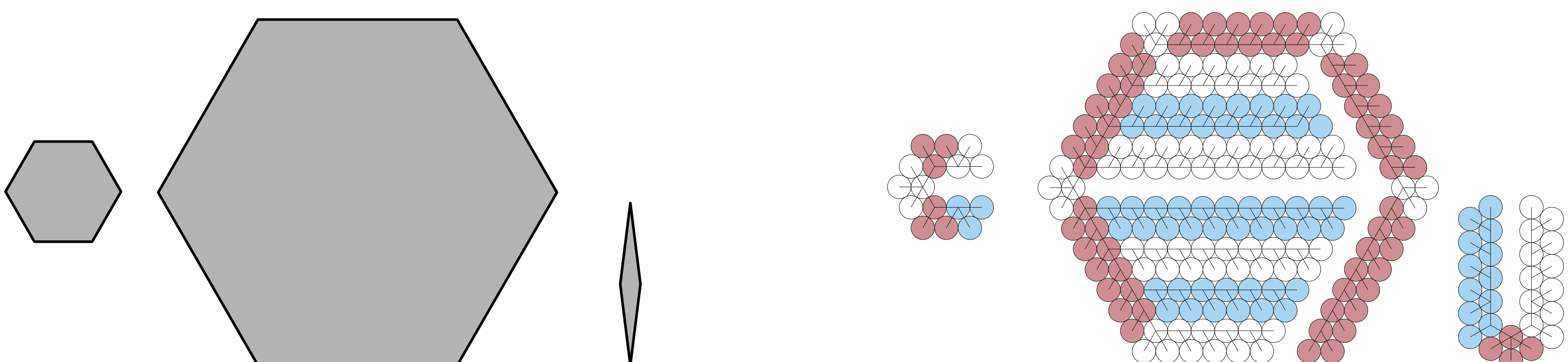

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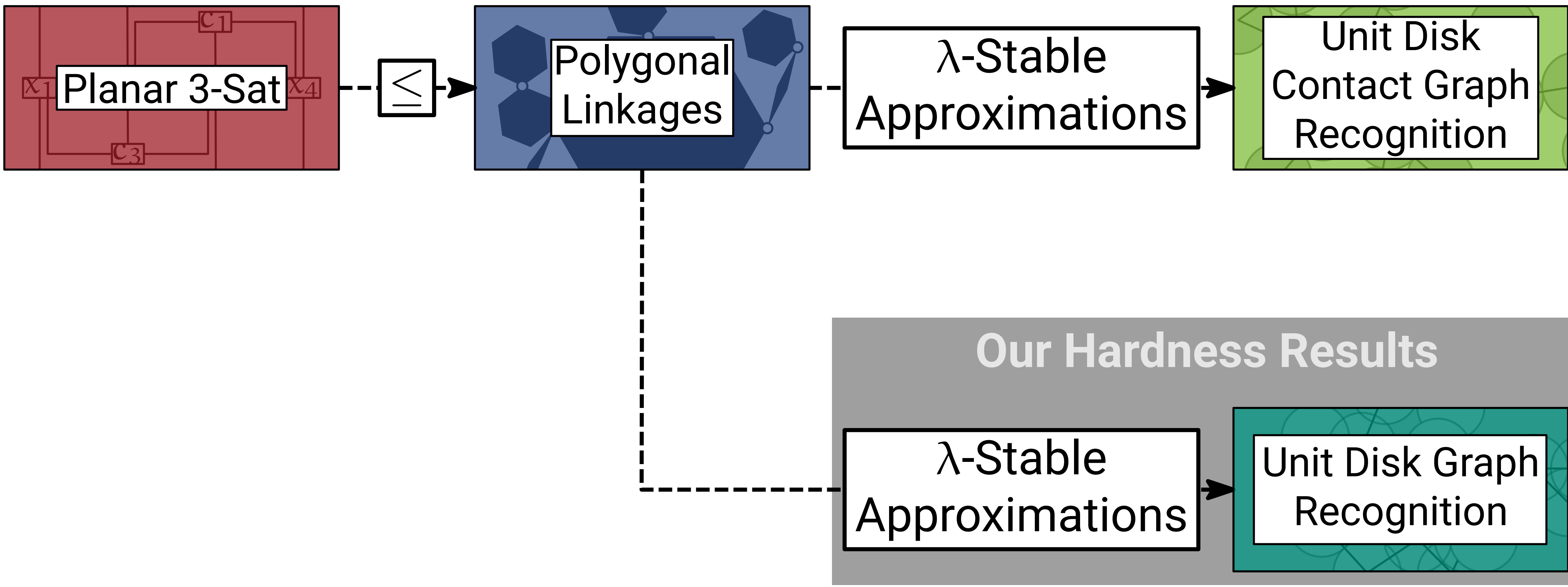

λ-Stable Approximations of Polygons

a **λ-stable approximation** of a polygon P

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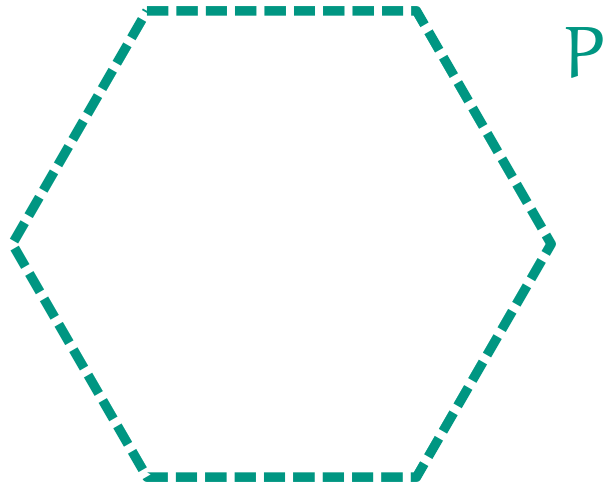

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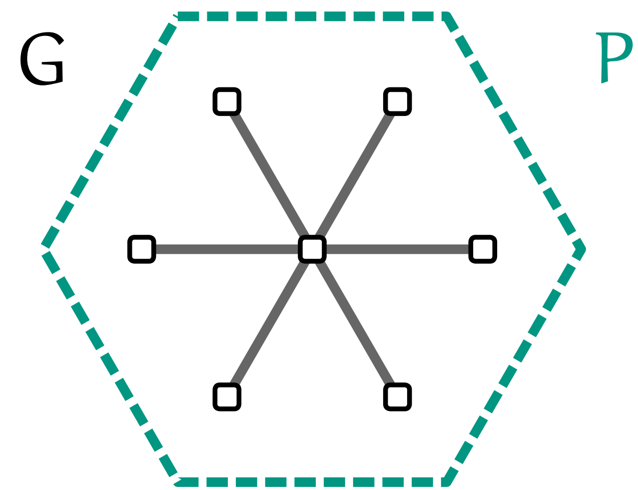

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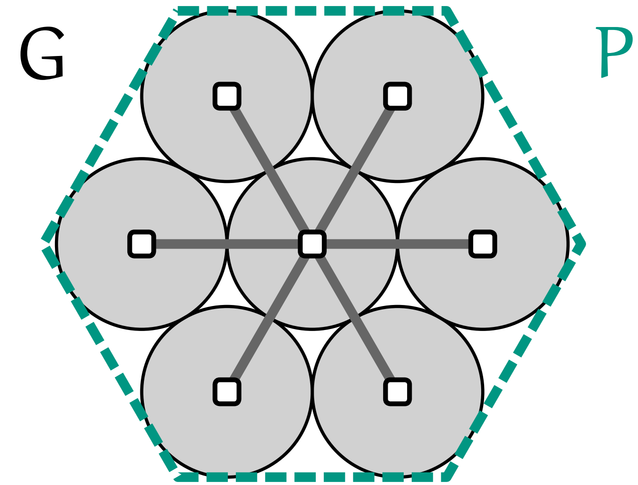

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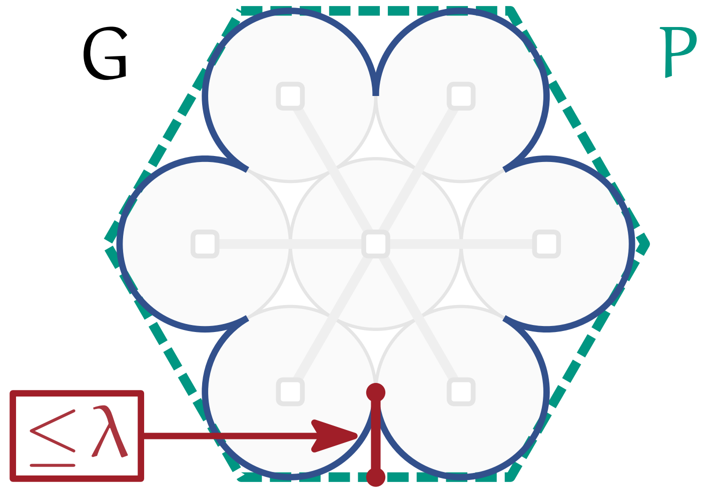

(directed) Hausdorff-Distance $\approx$ How far do you have to travel from any point on P to reach the UDC boundary of G

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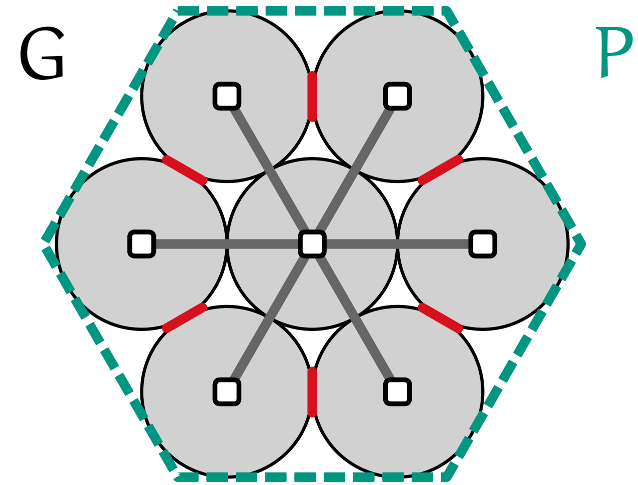

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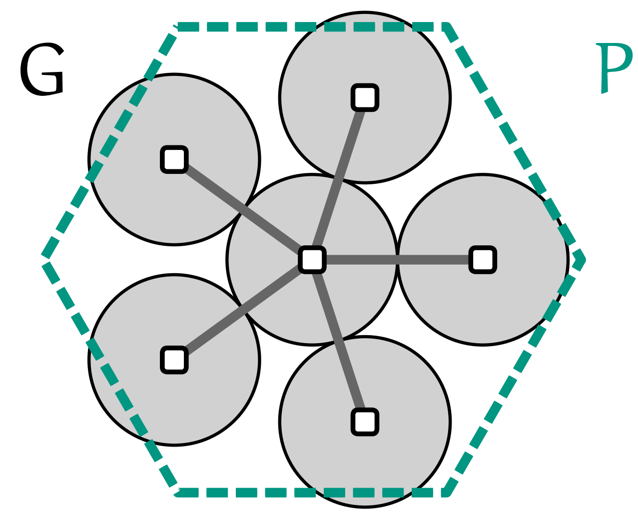

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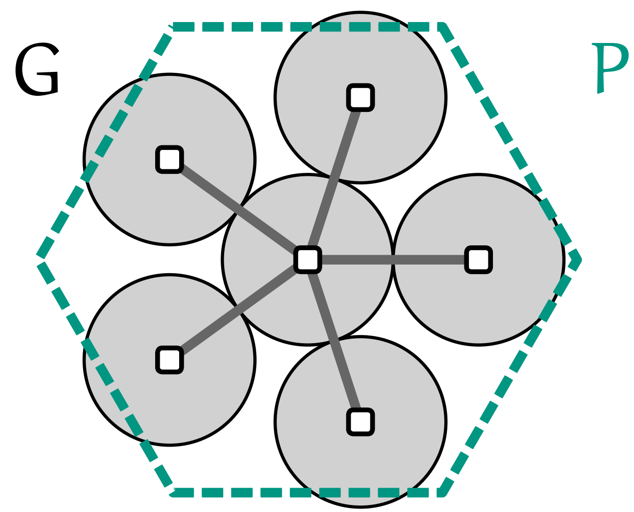

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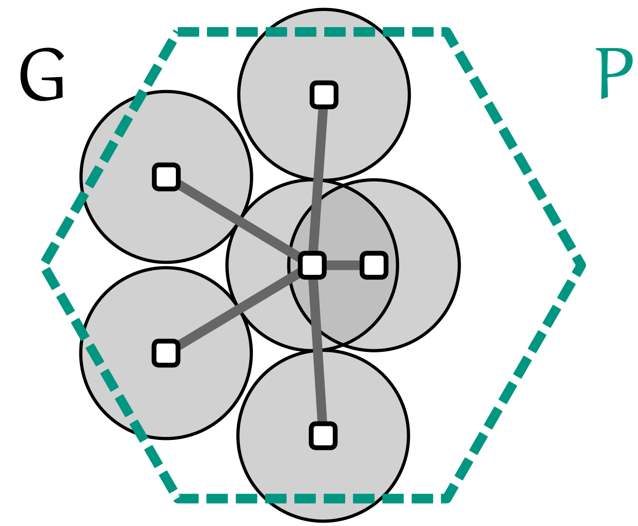

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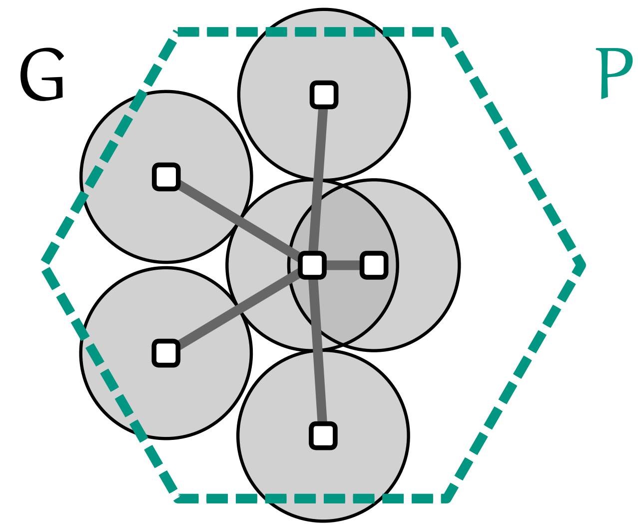

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- UDR allow for overlap
- Must hold for all UDRs

$\Longrightarrow$ We have to force a very restricted placement in the UDR.

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What about tracing a straight line?

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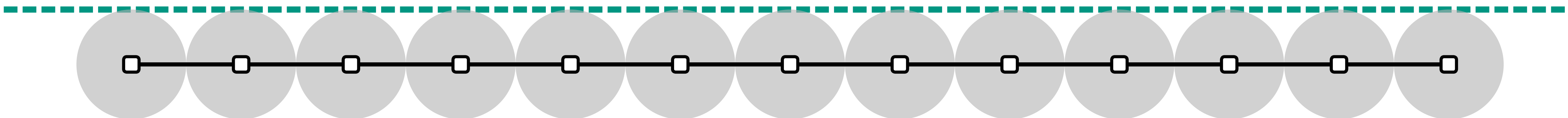

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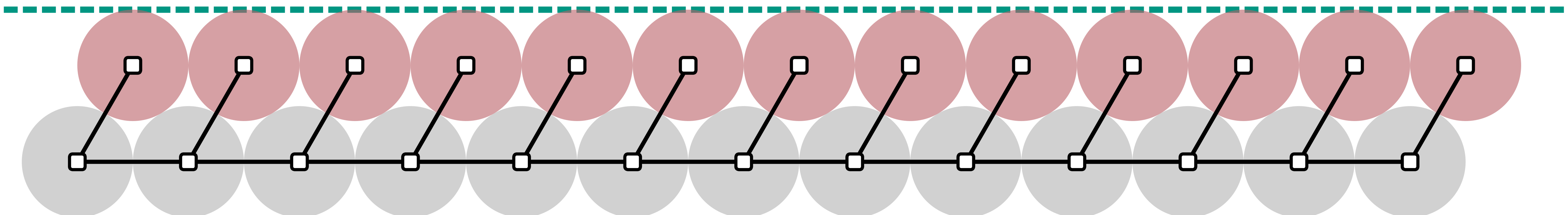

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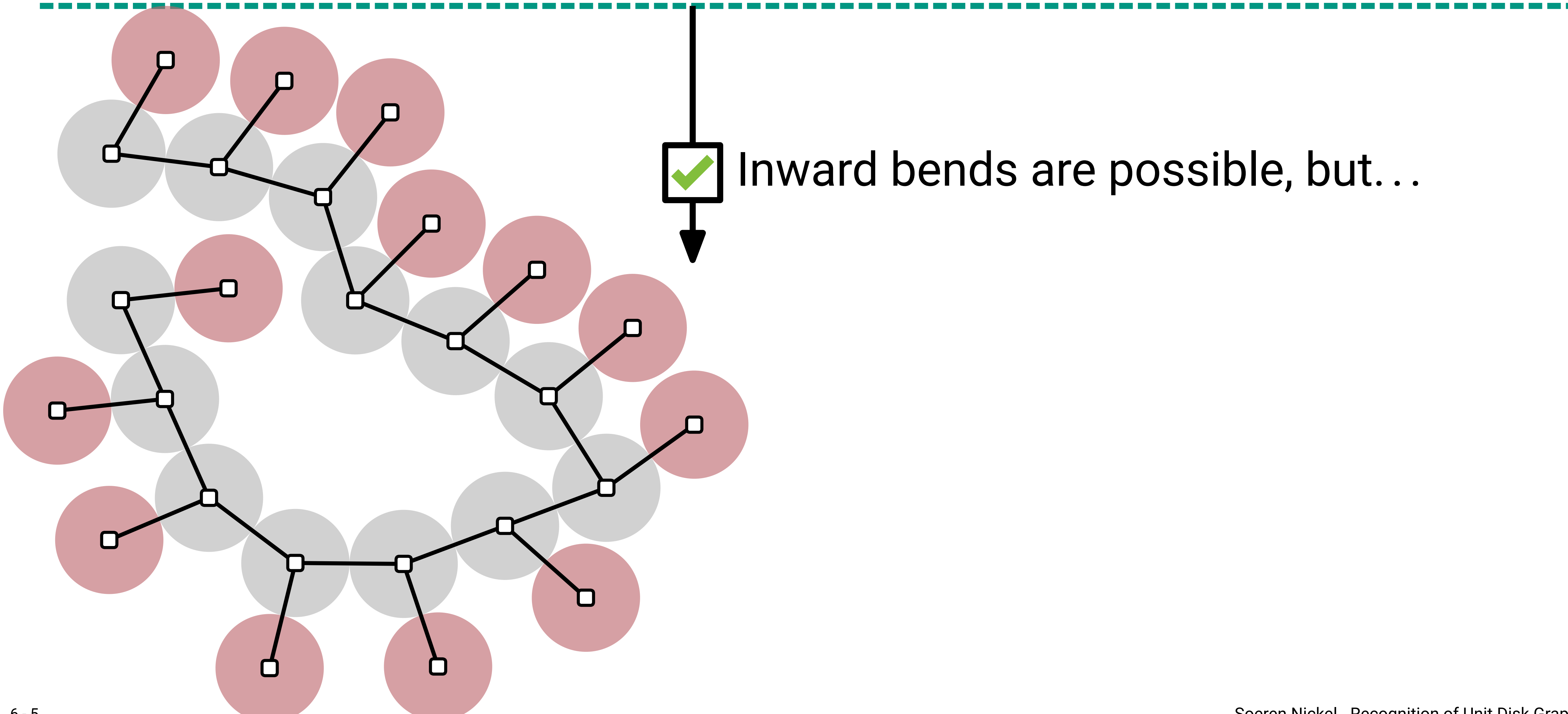

Inward bends are possible, but...

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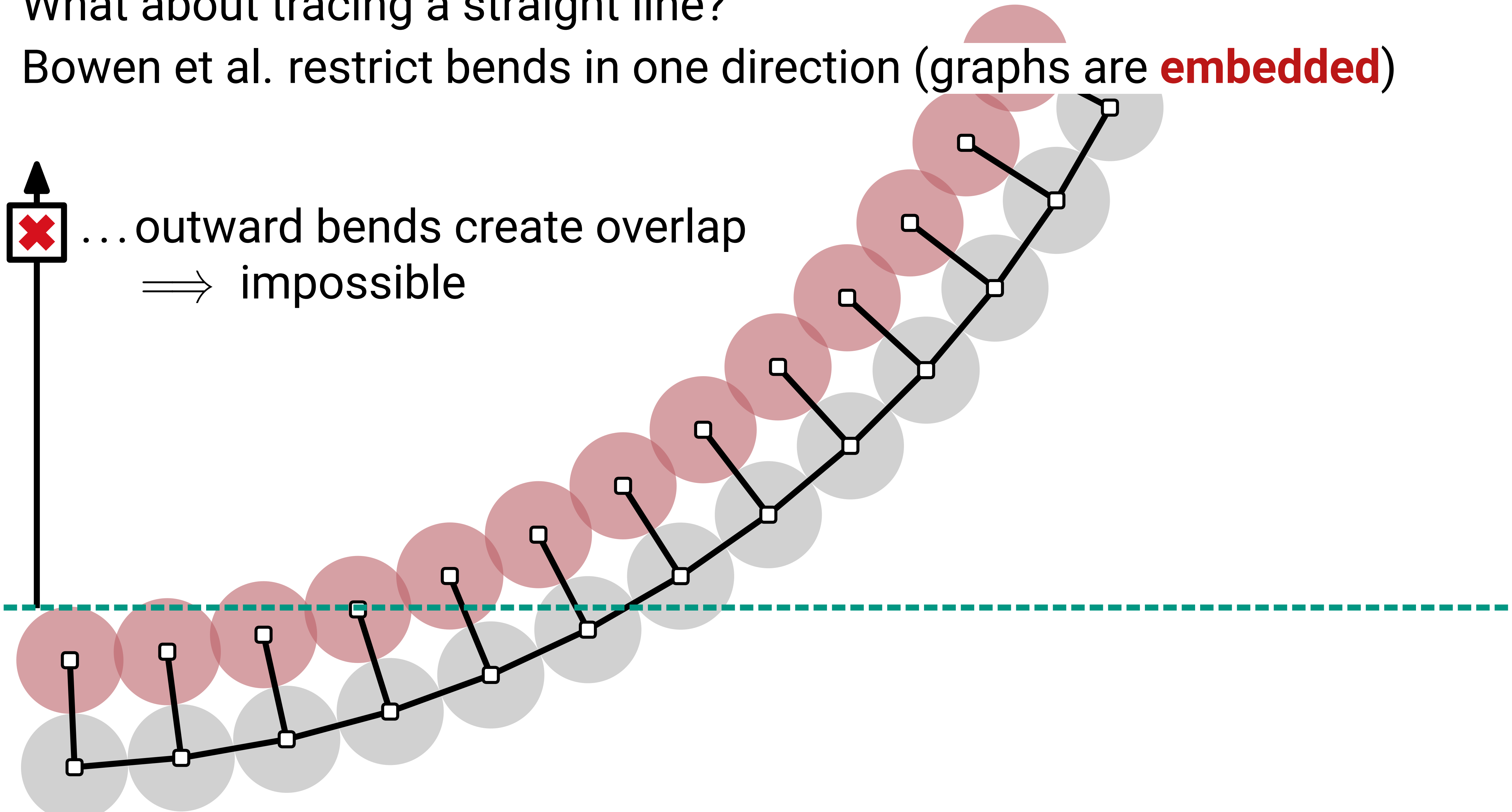

. . . outward bends create overlap

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Bend Restrictions in Embedded UDRs

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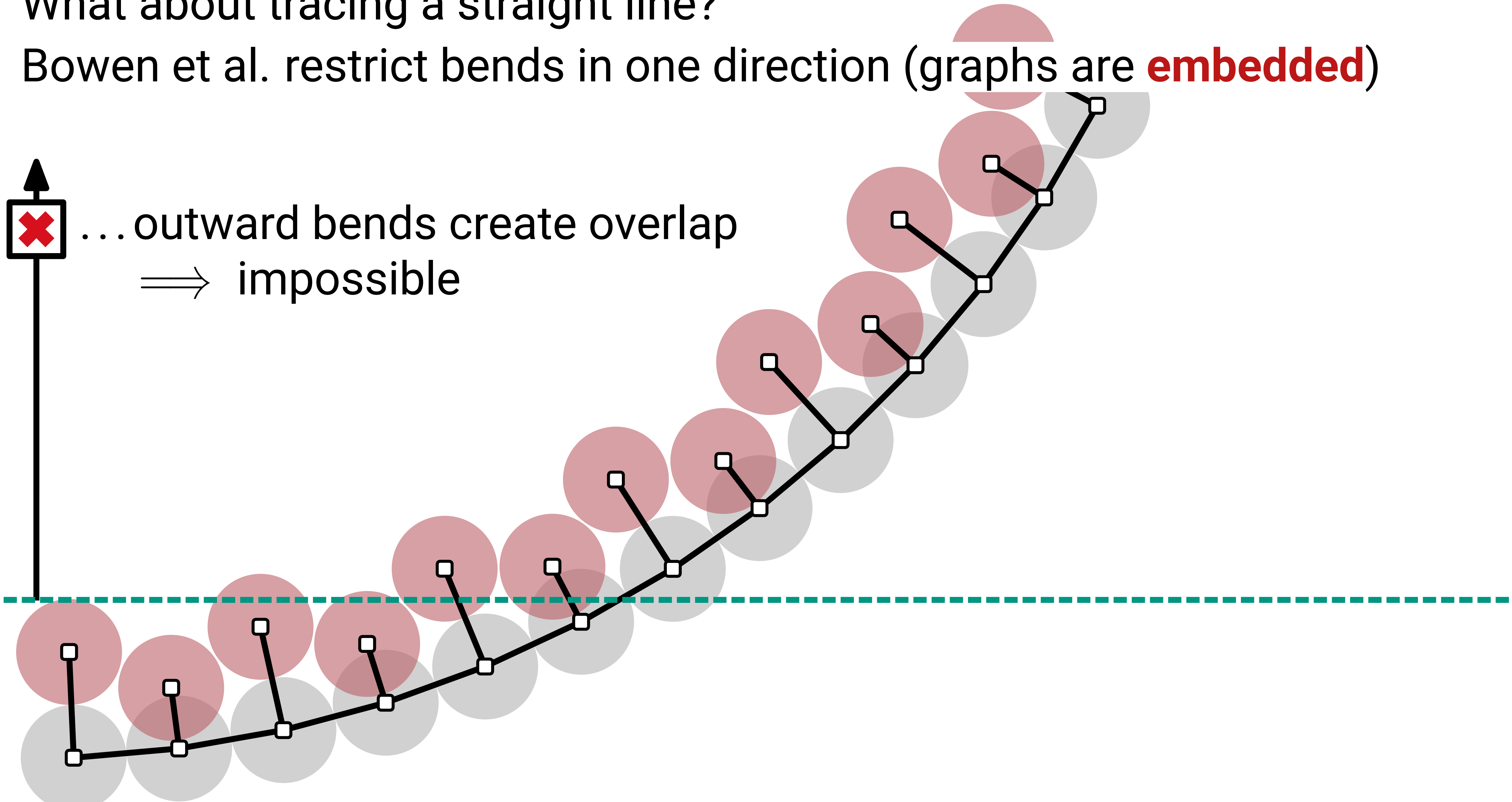

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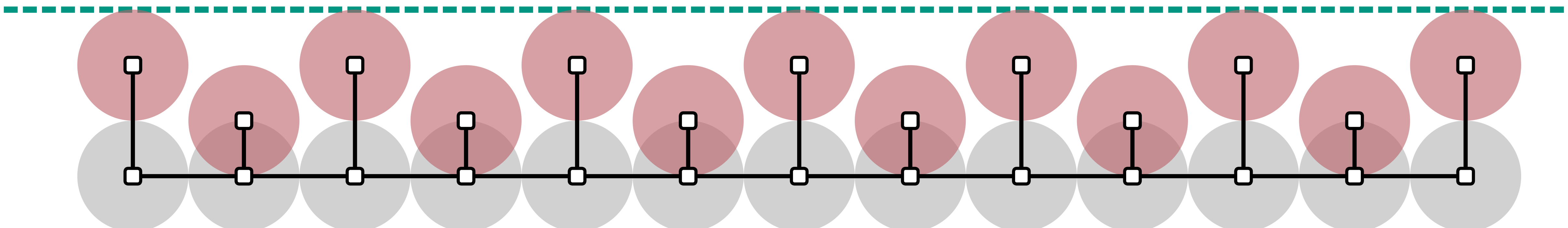

Does not work immediately in a **UDR**

Overlaps create space for outward bends

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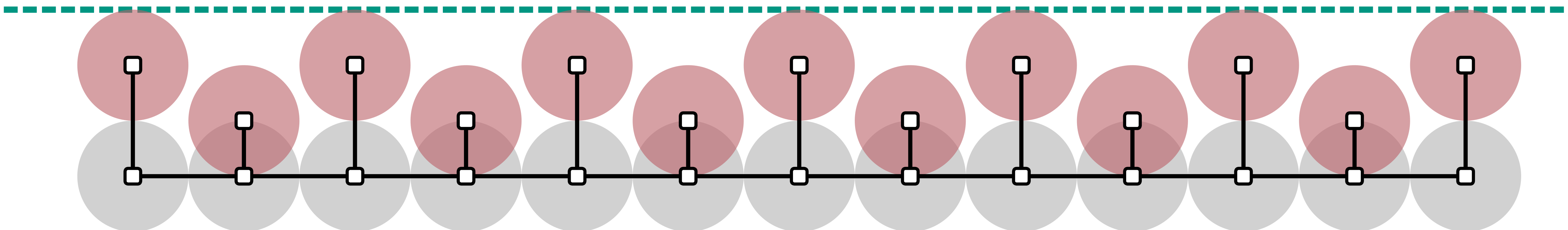

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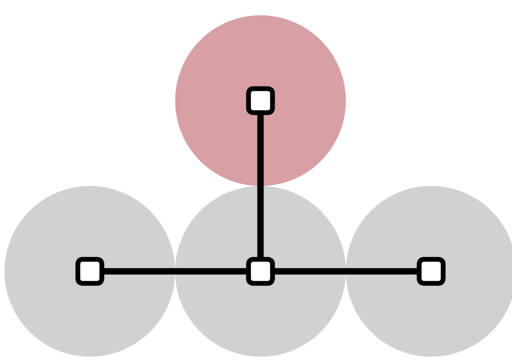

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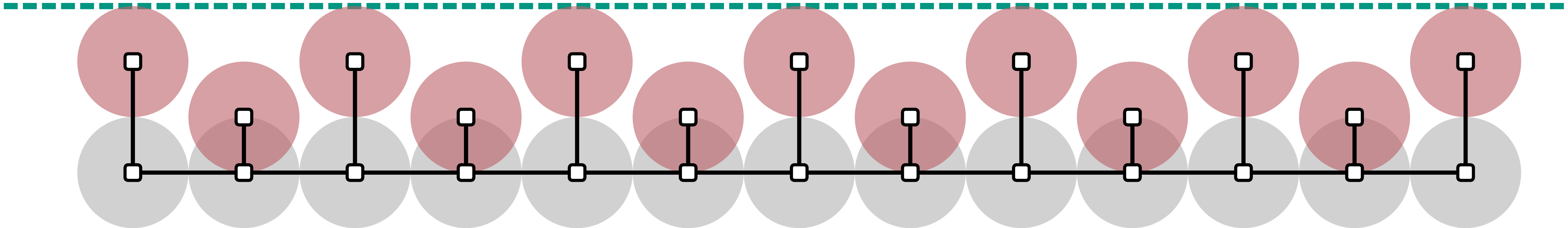

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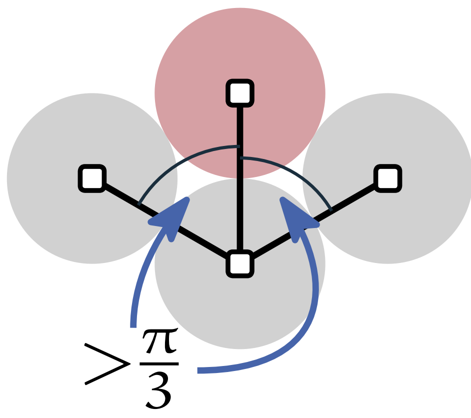

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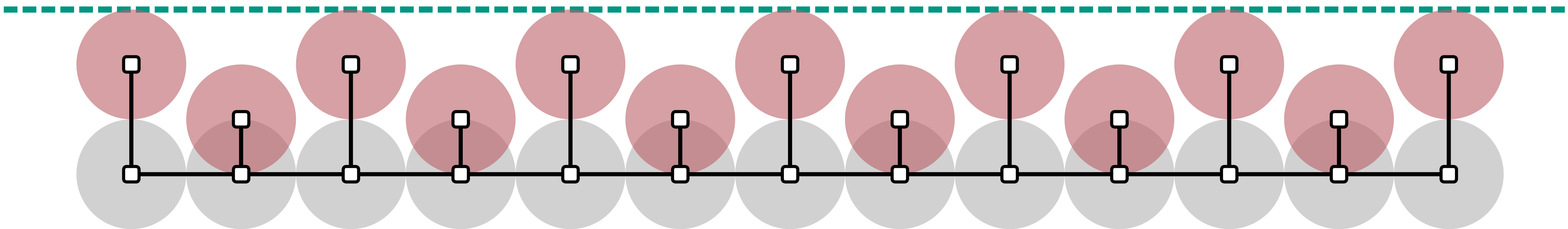

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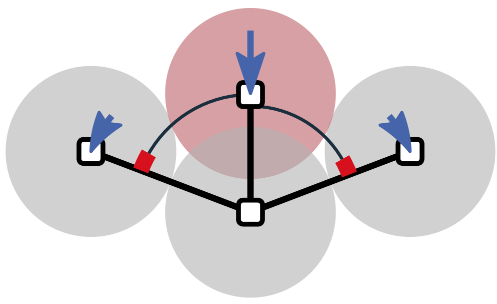

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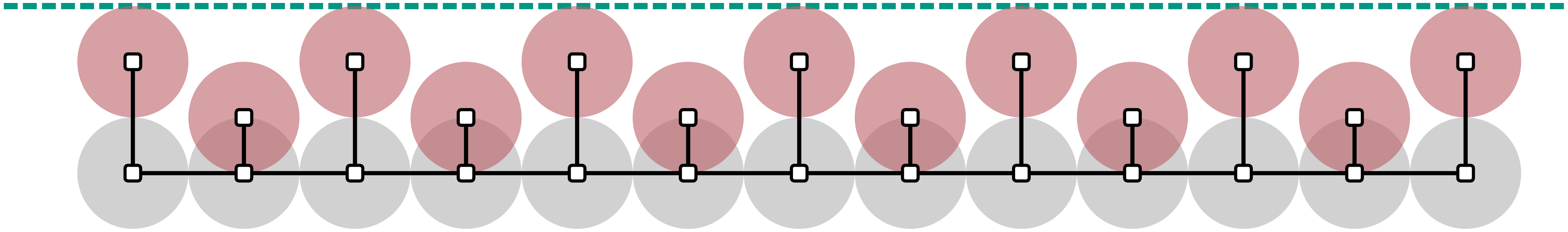

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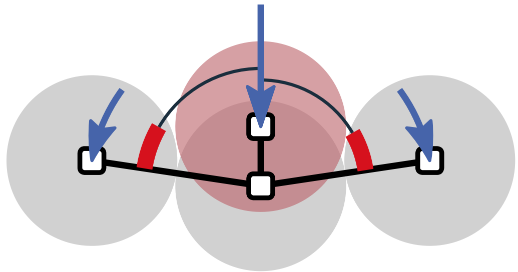

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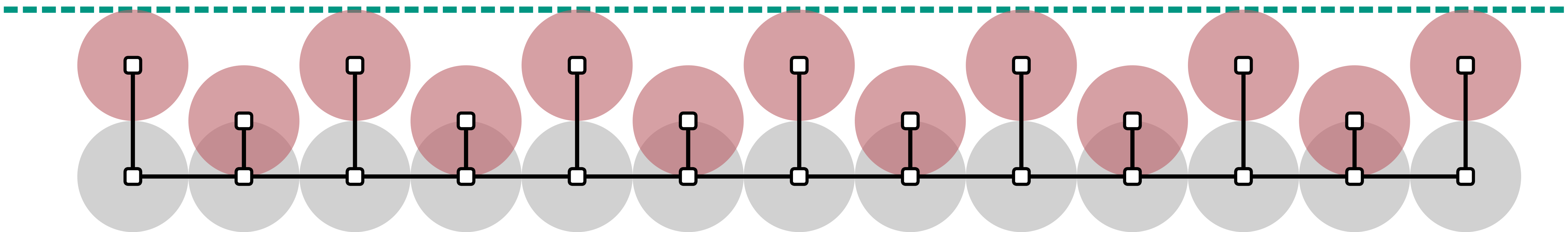

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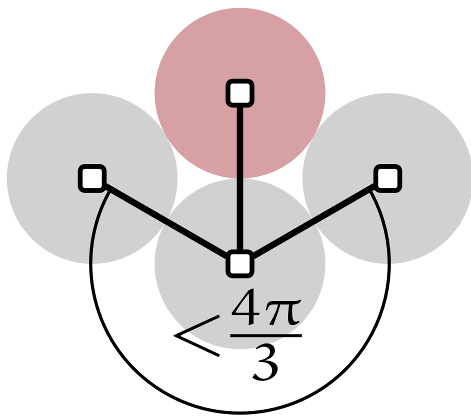

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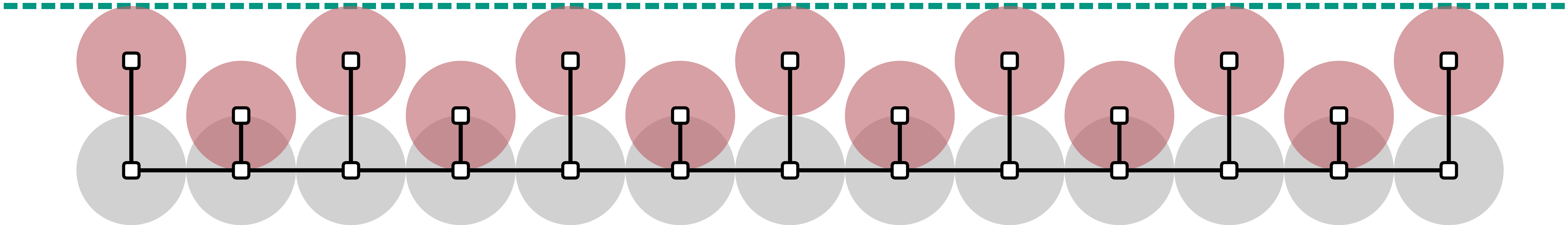

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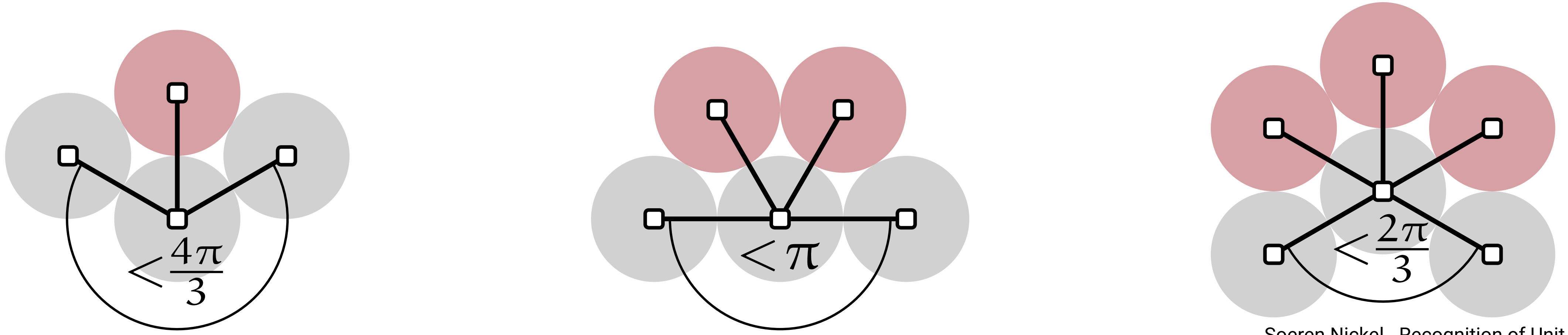

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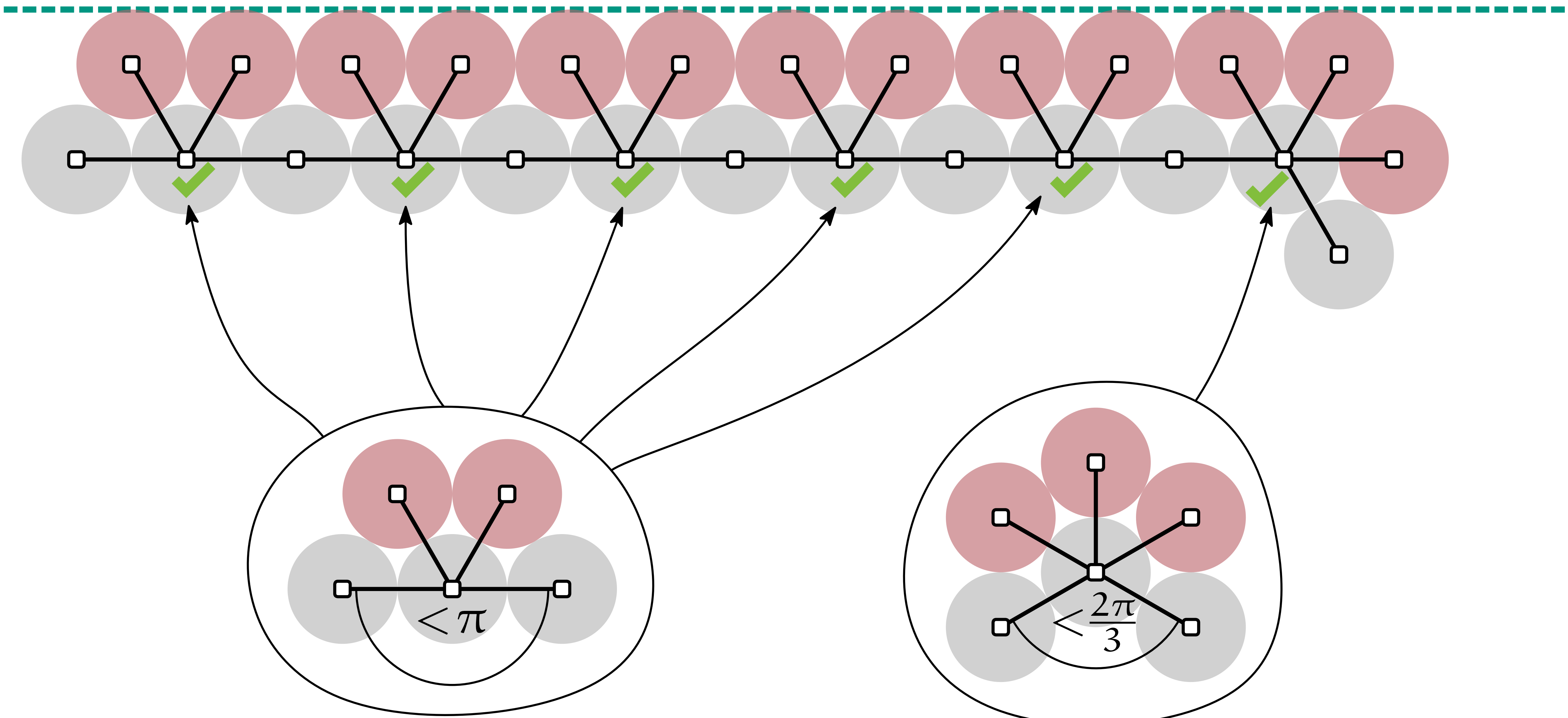

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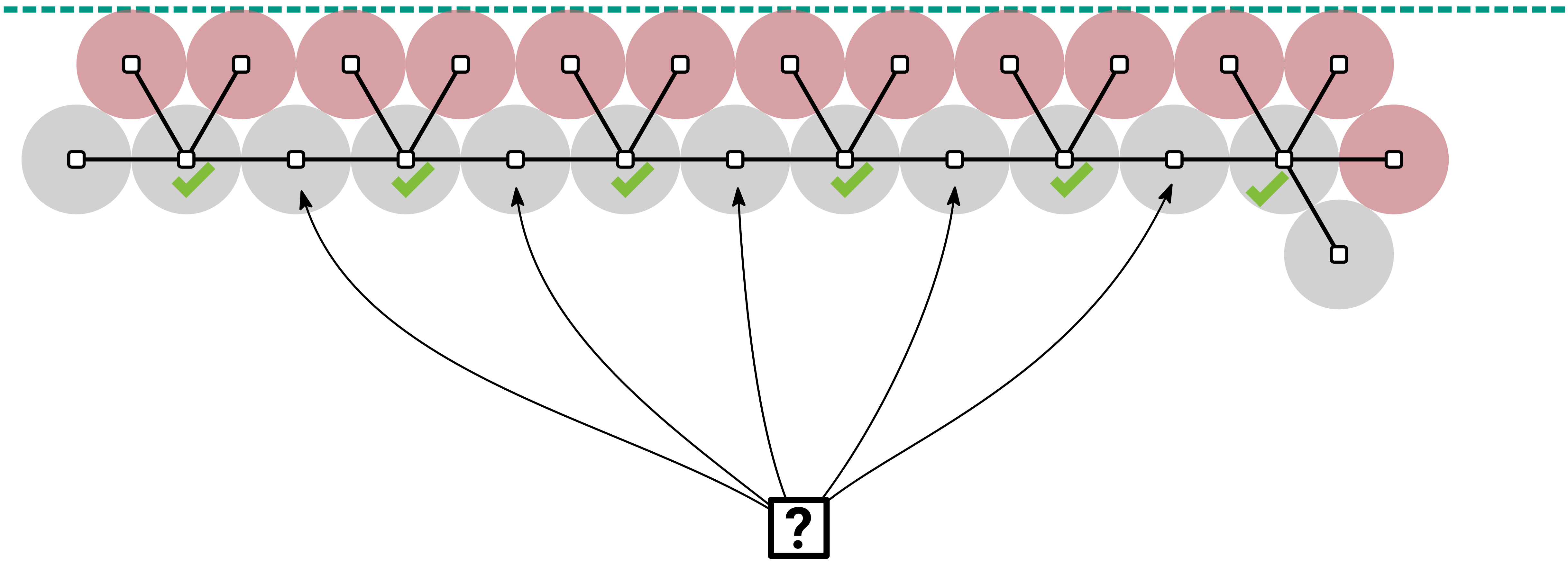

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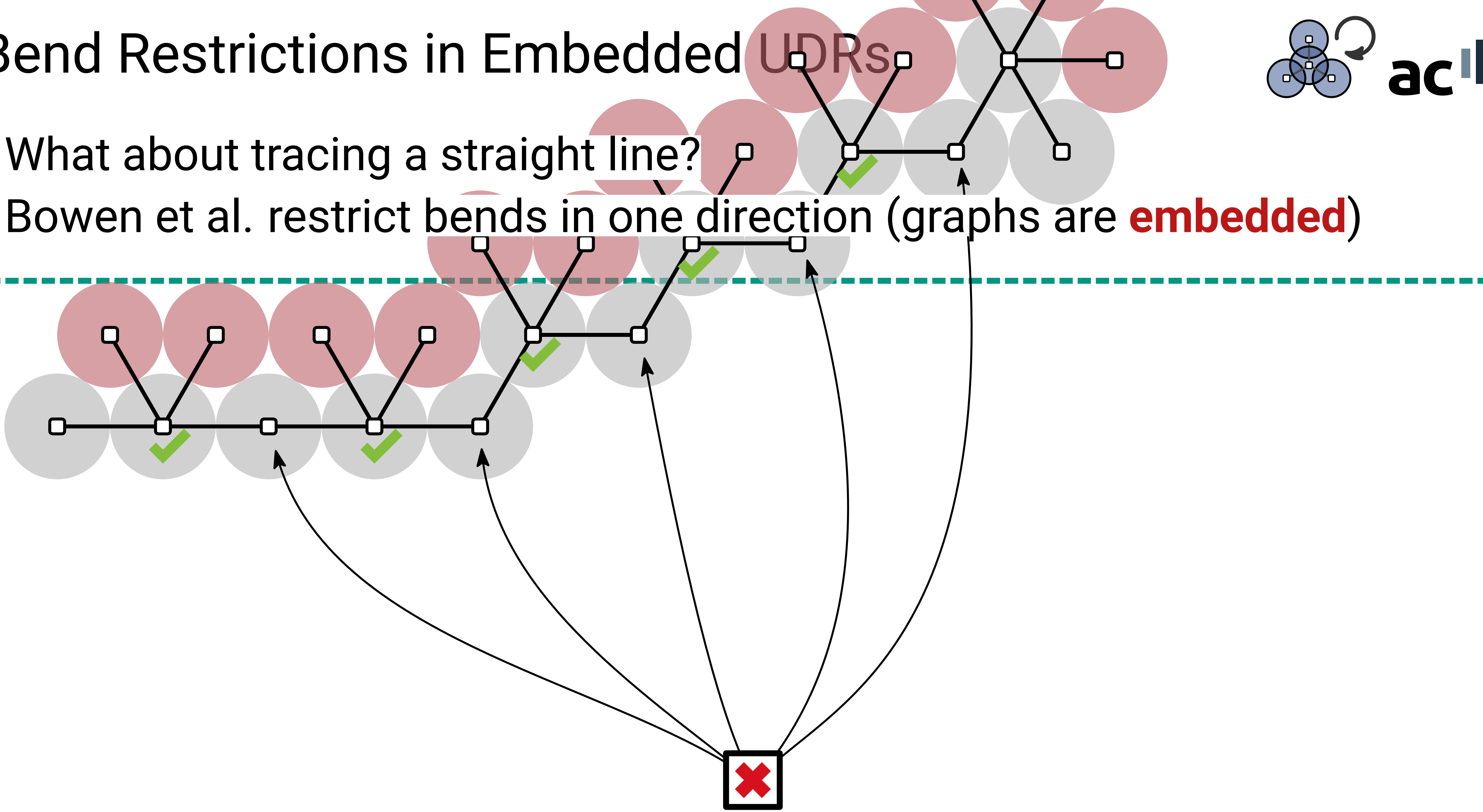

We need additional restricting neighbors

Bend Restrictions in Embedded UDRs

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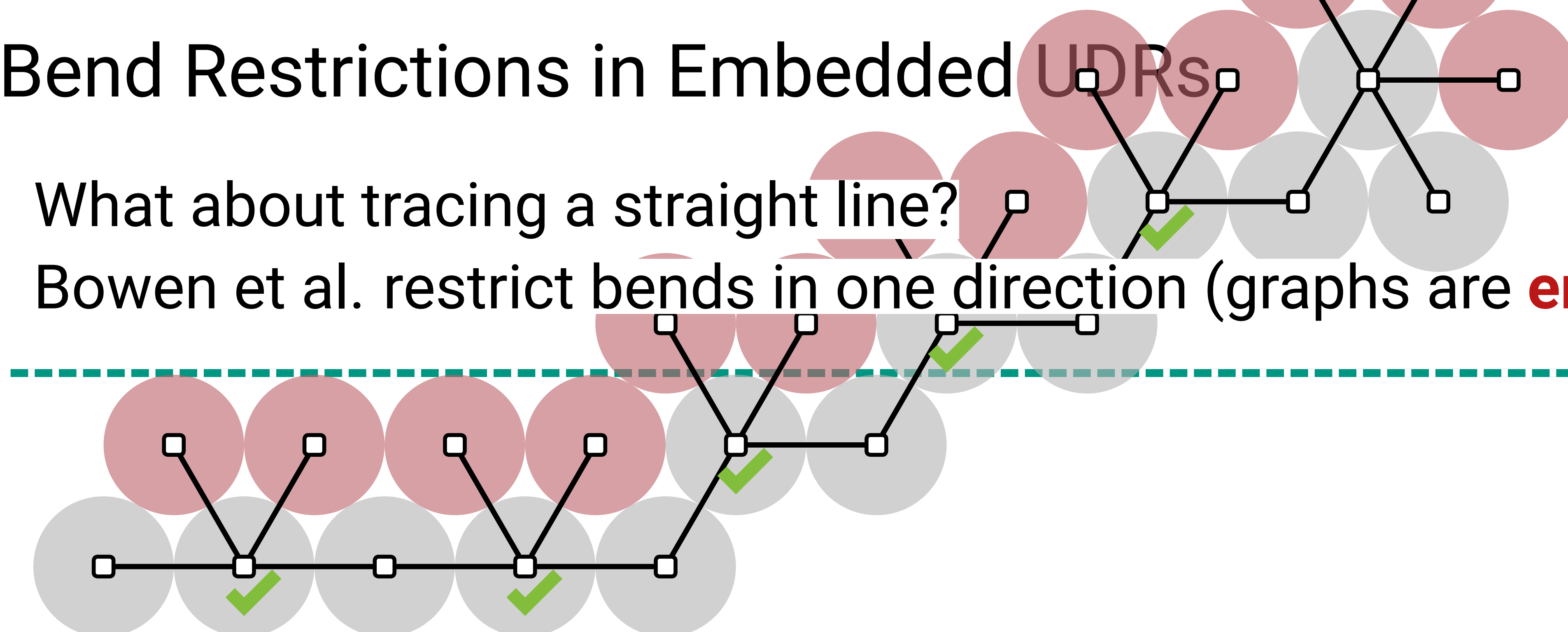
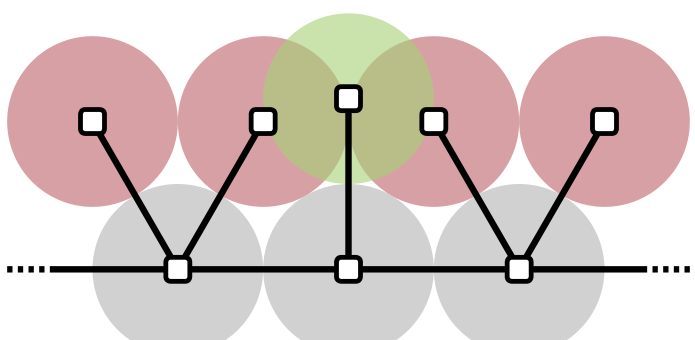


We can force nearly complete overlap with a **single disk** at these nodes



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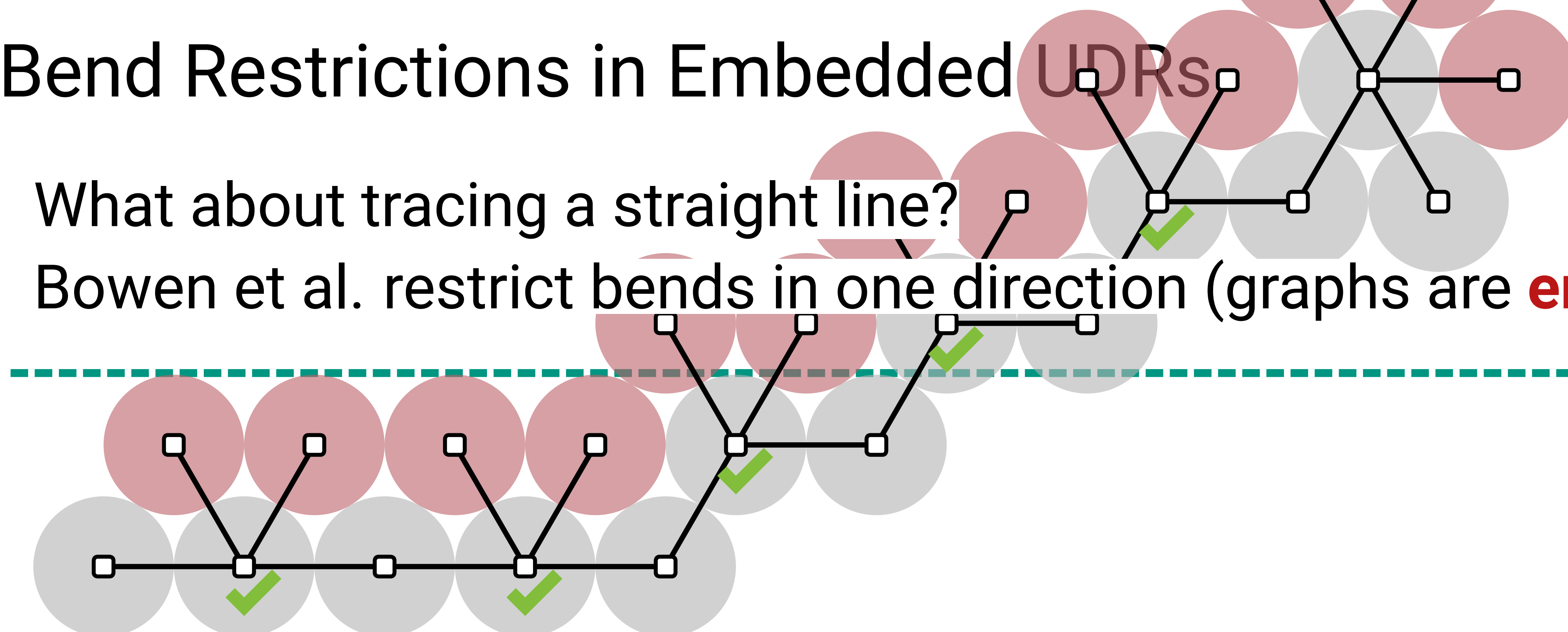

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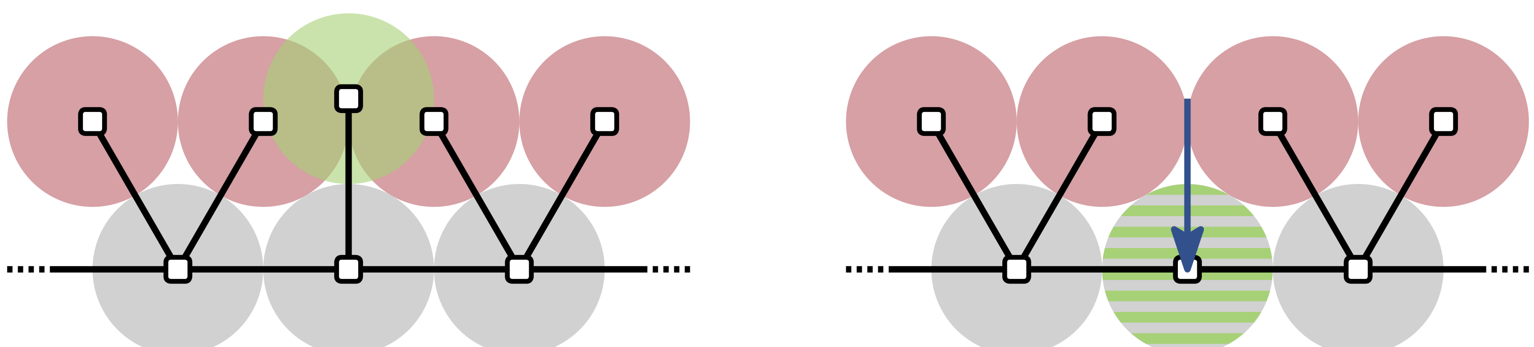

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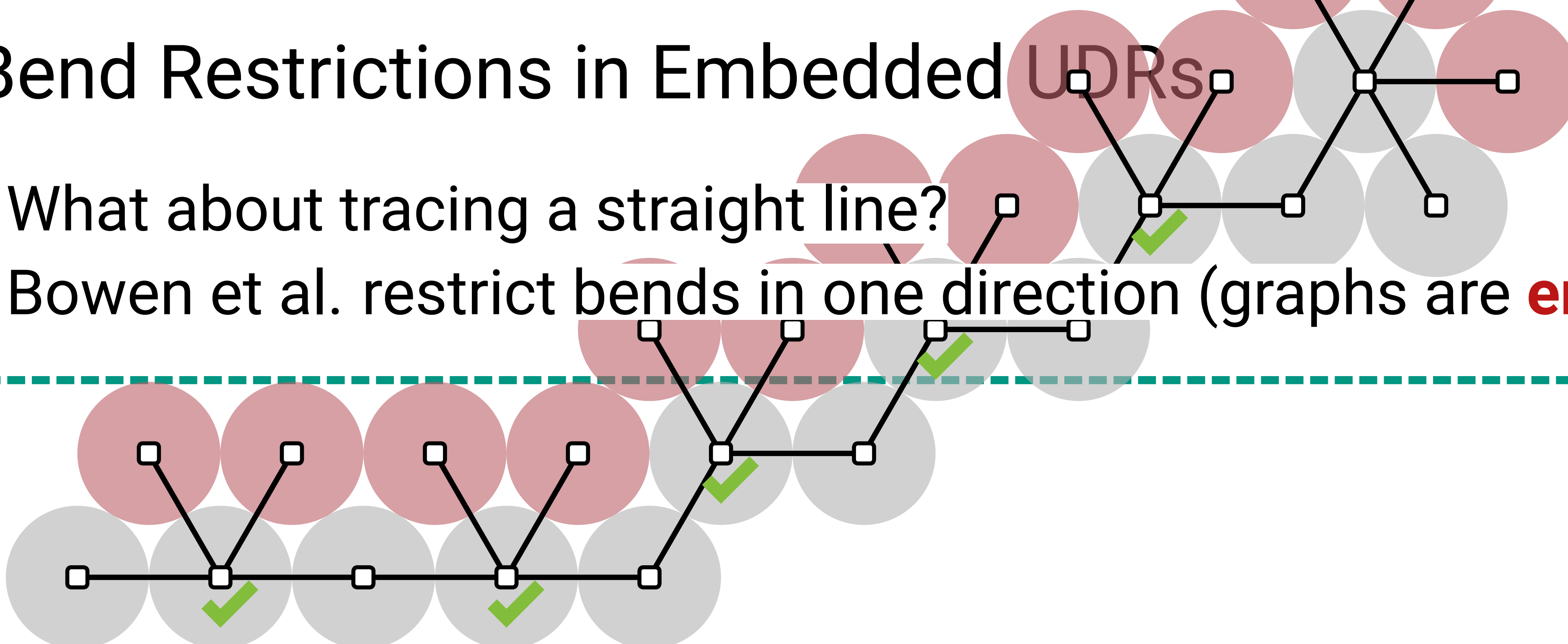

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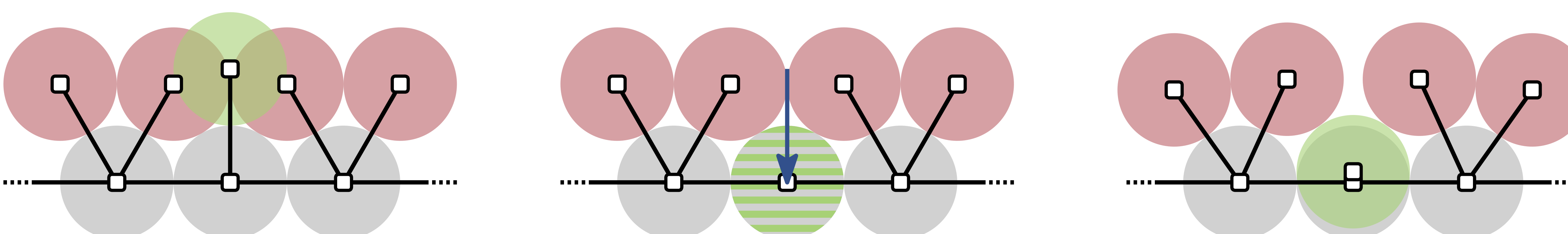

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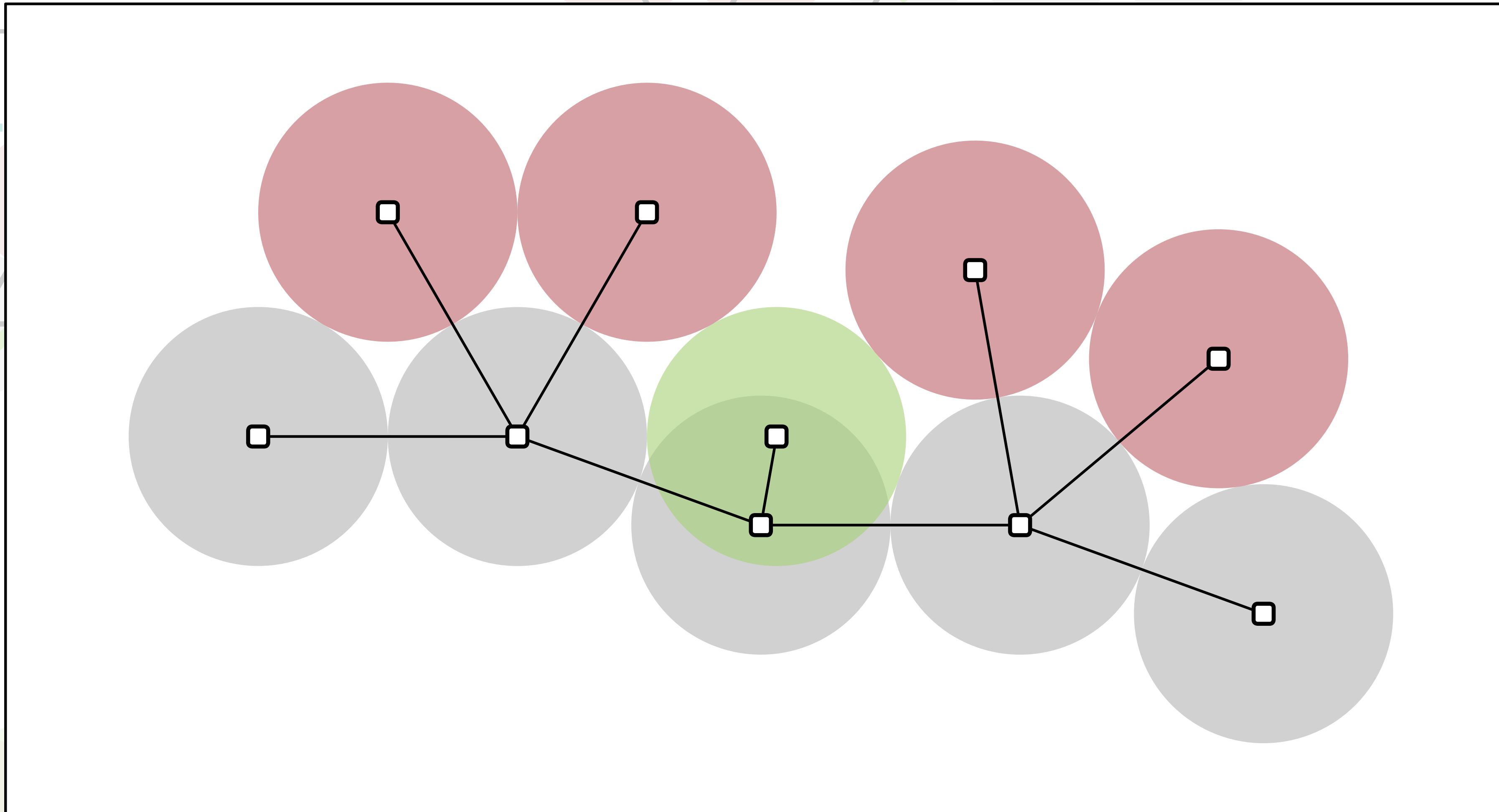

We can fo nodes

Bend Restrictions in Embedded UDRs

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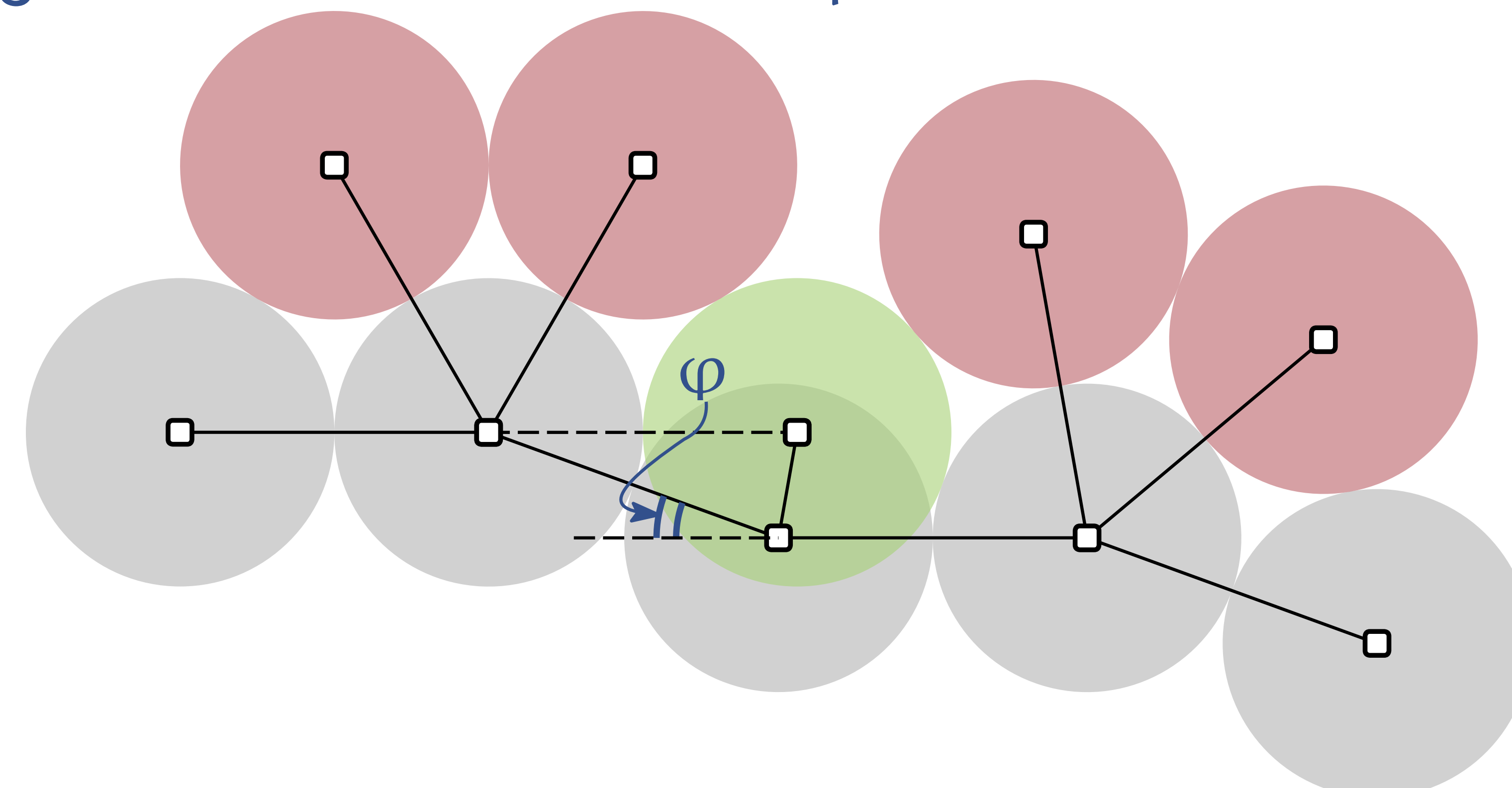
We can fo ... nodes

To gain an outward bend of φ...



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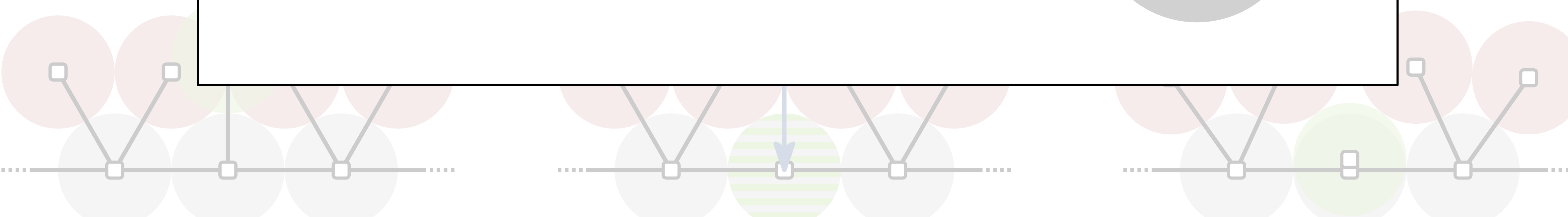

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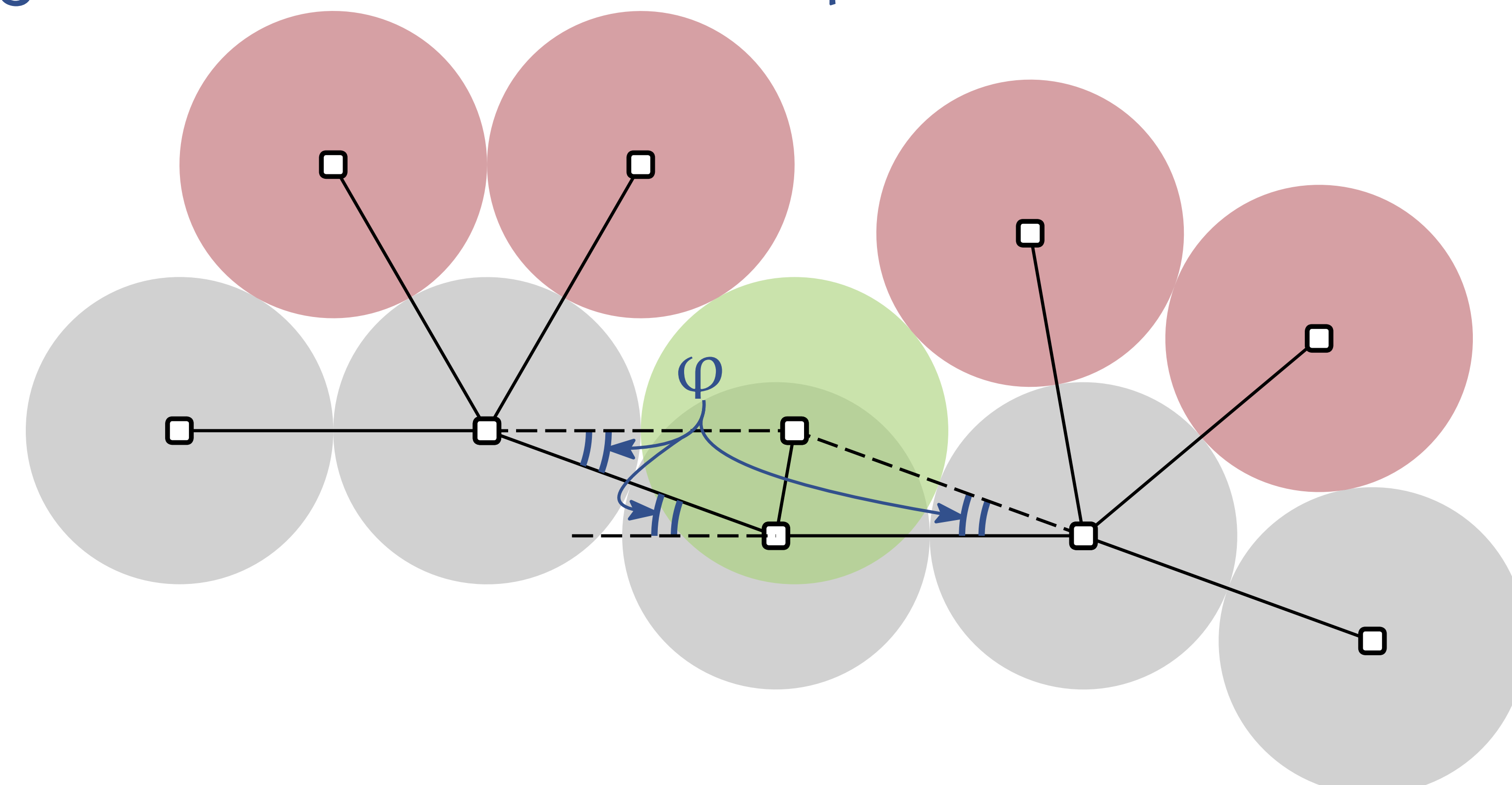

Bend Restrictions in Embedded UDRs

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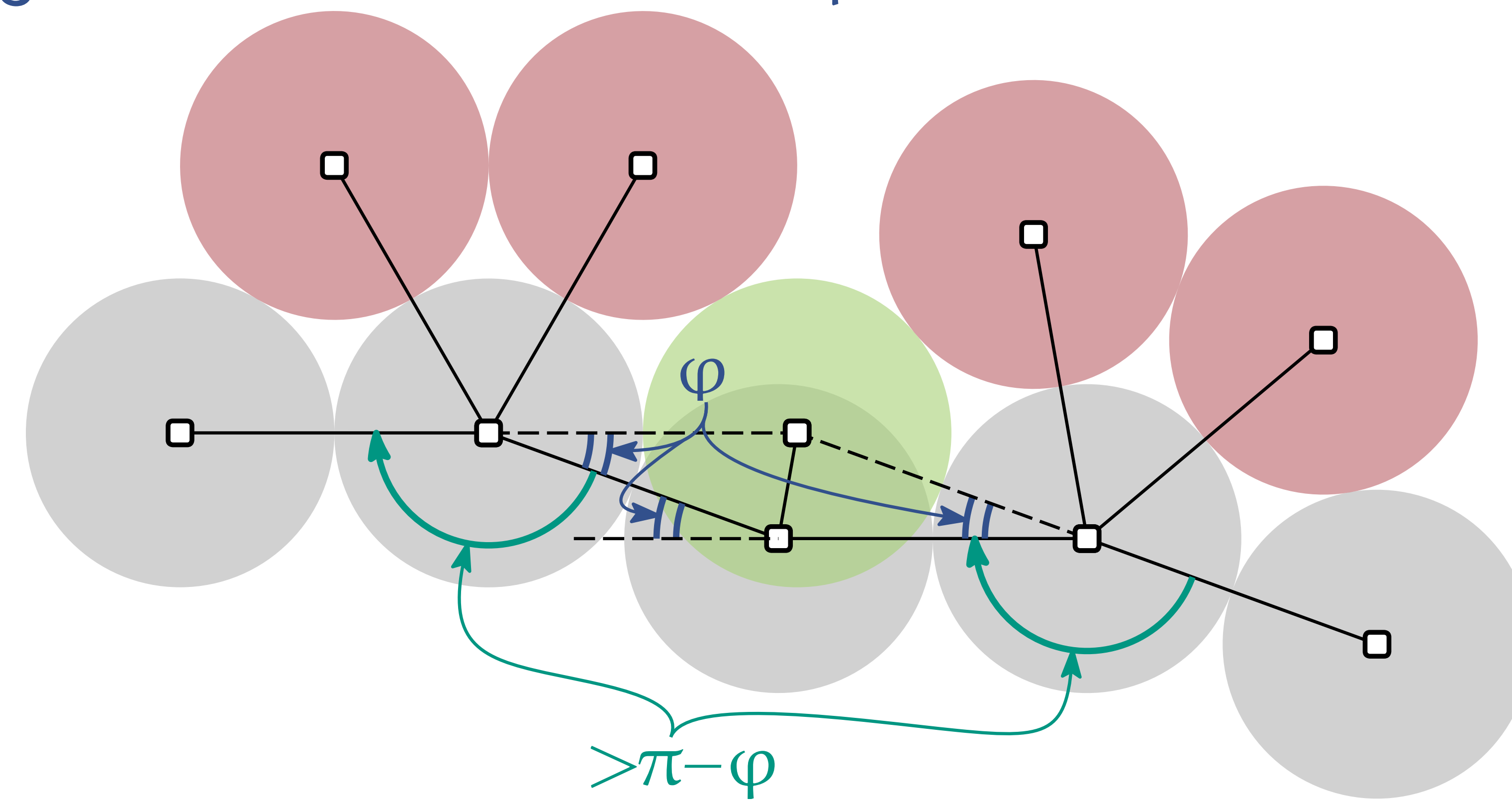
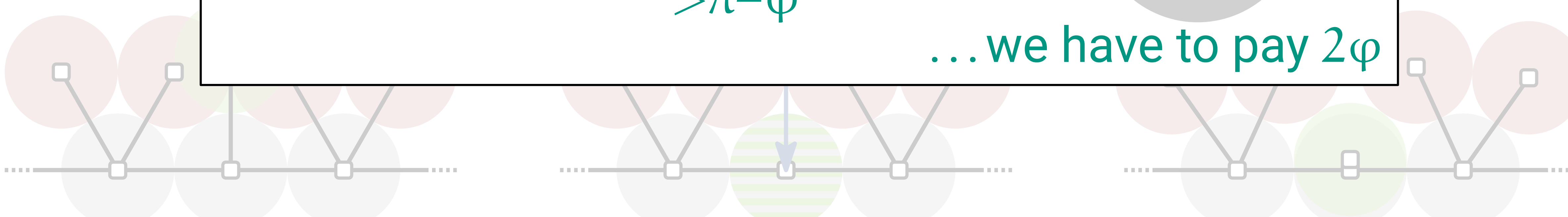


…we have to pay 2φ

We can fo… …nodes



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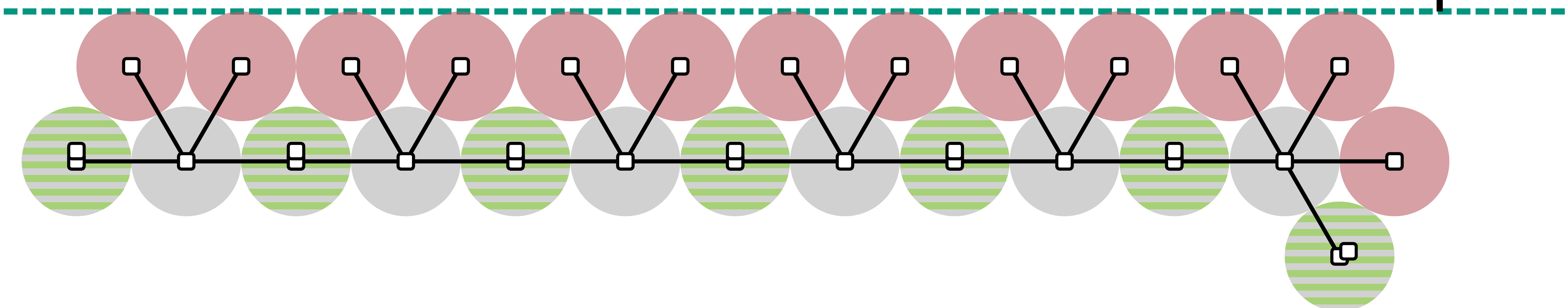

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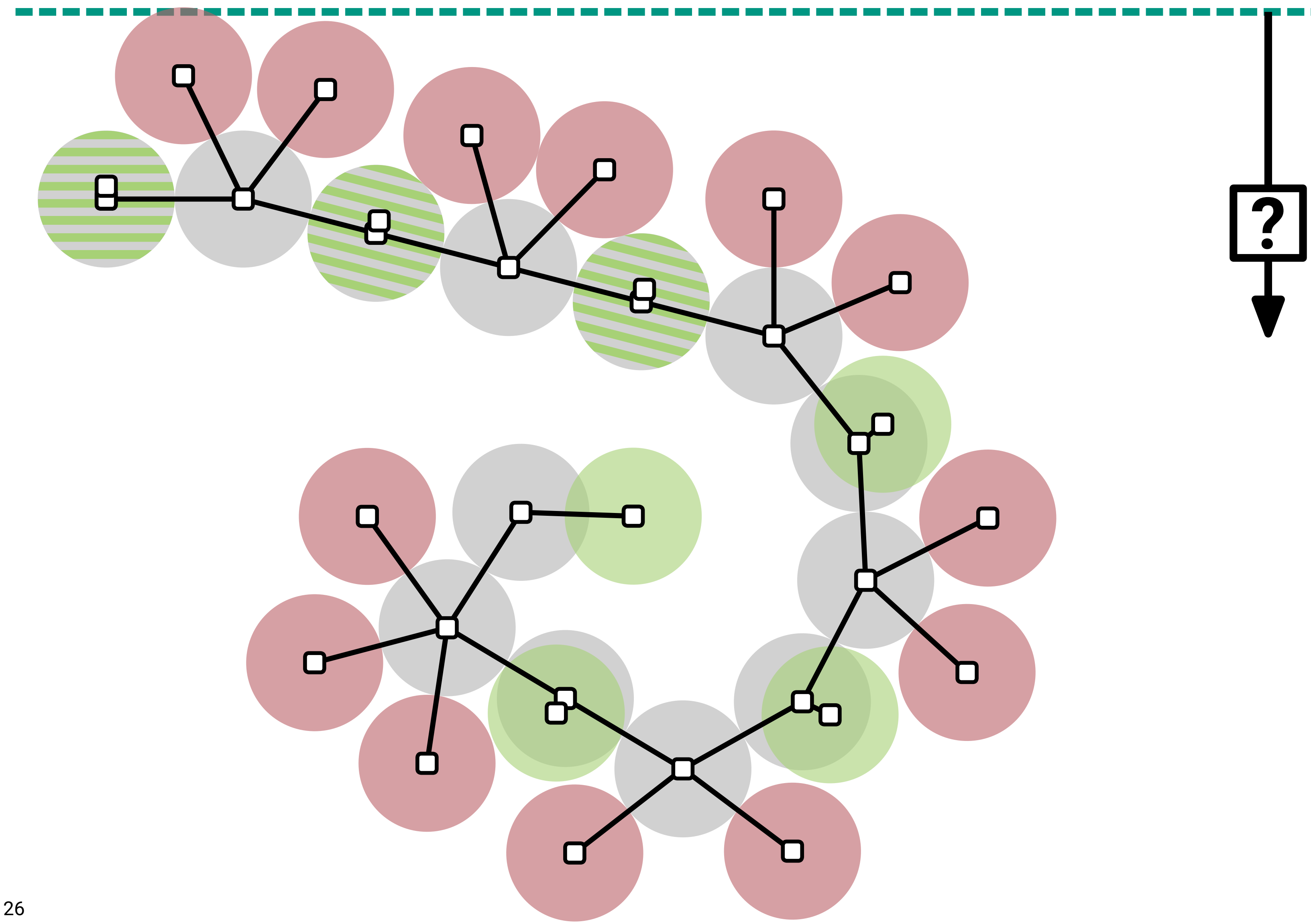

How do we treat inward flexibility?

Bend Restrictions in Embedded UDRs

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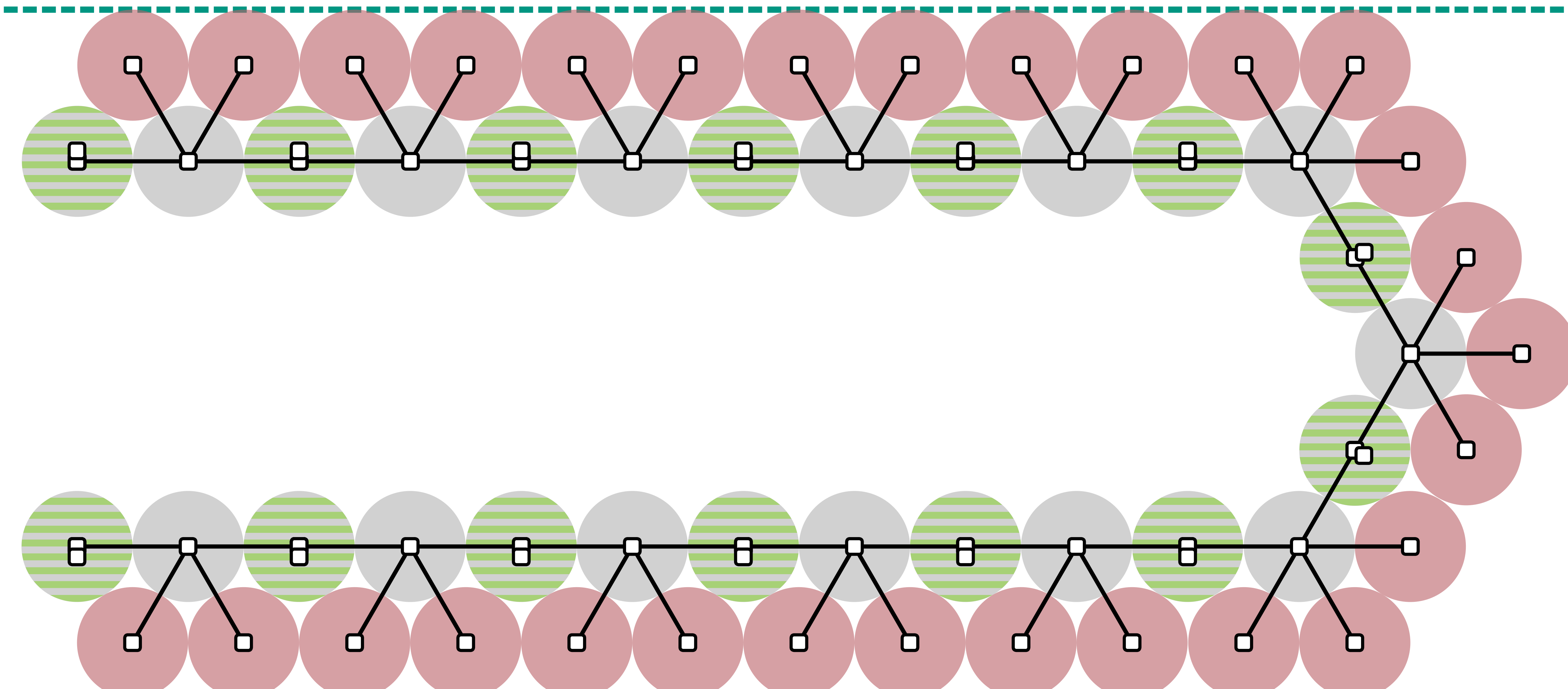

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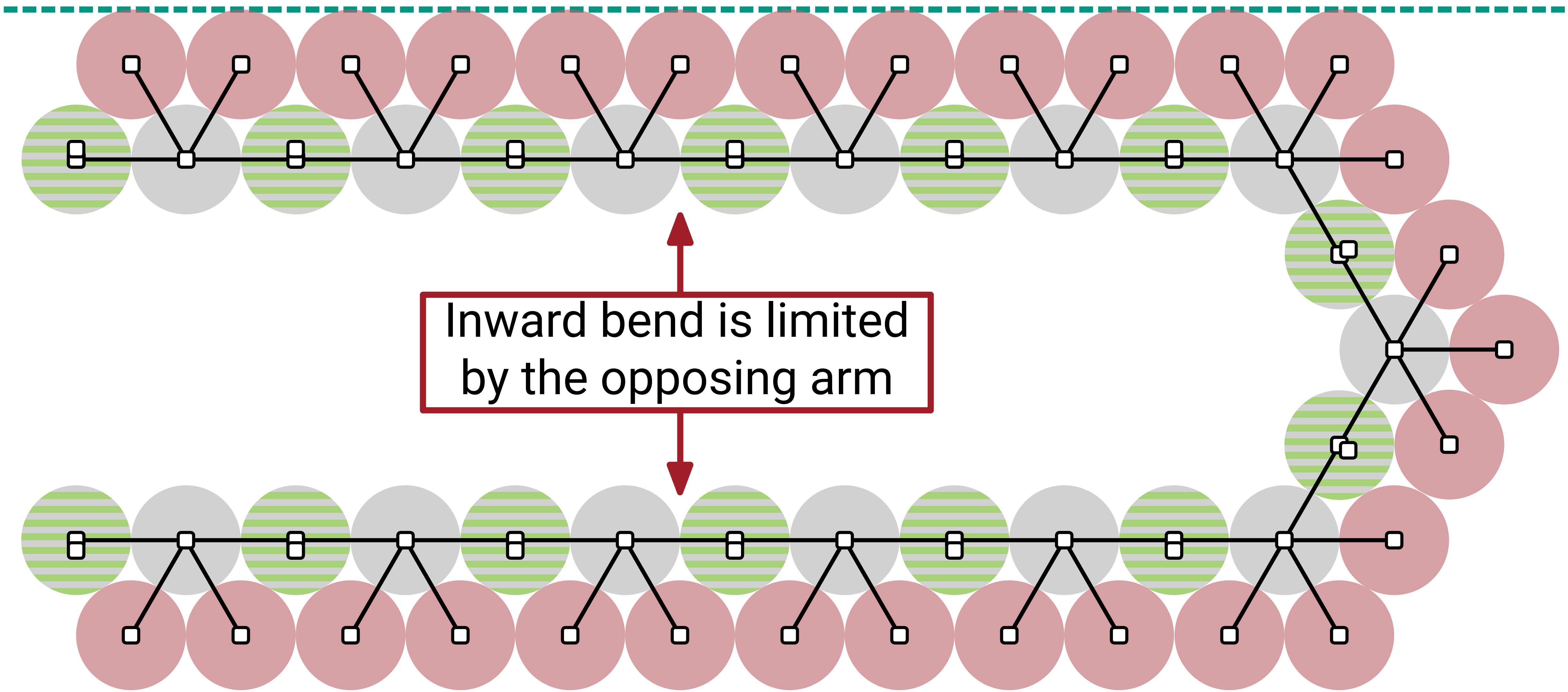

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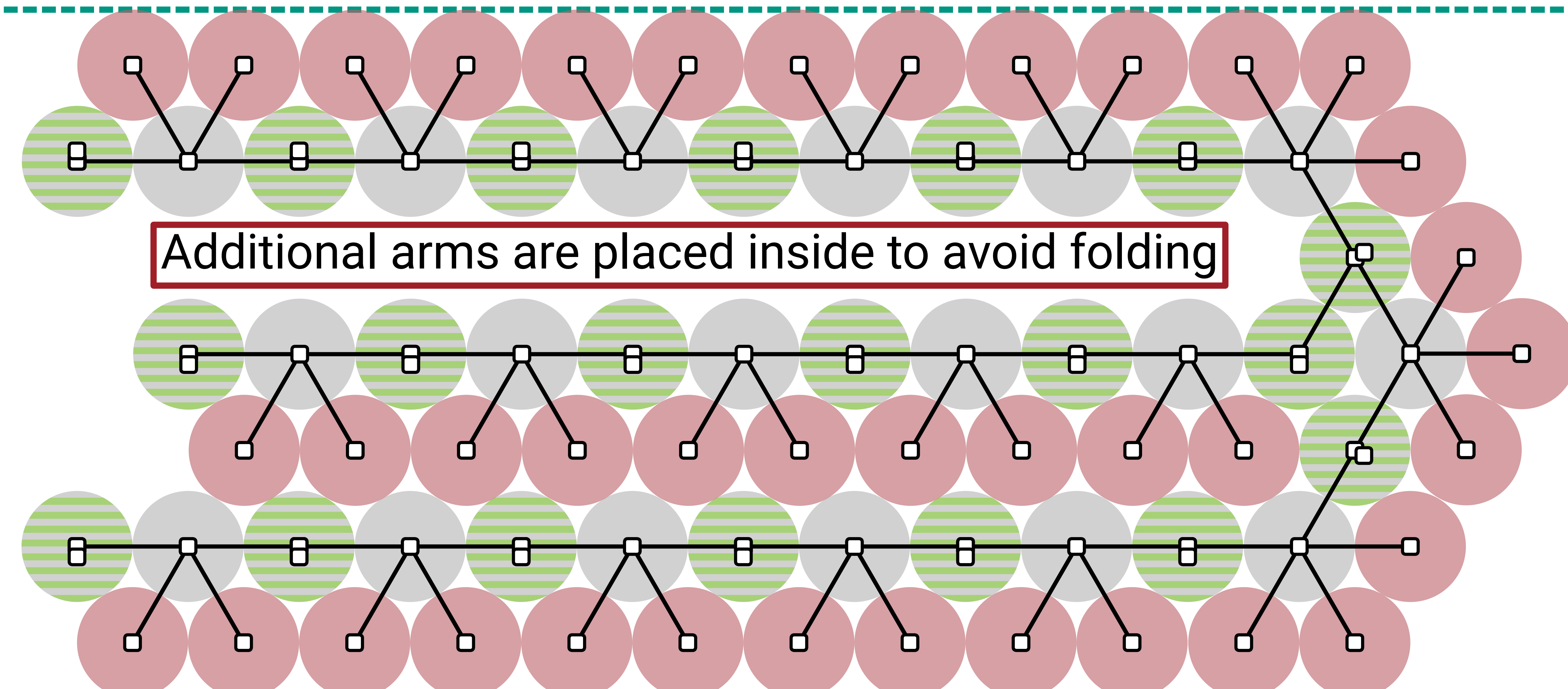

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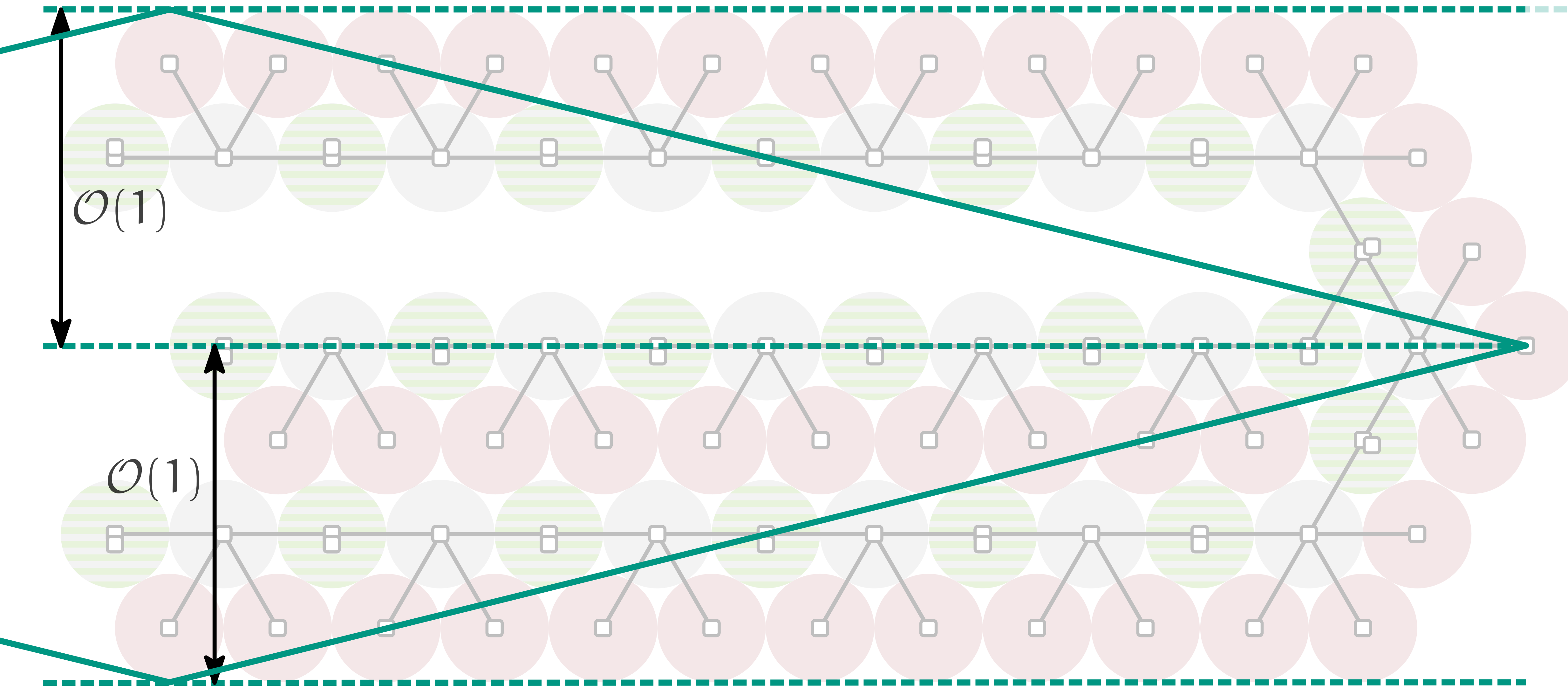
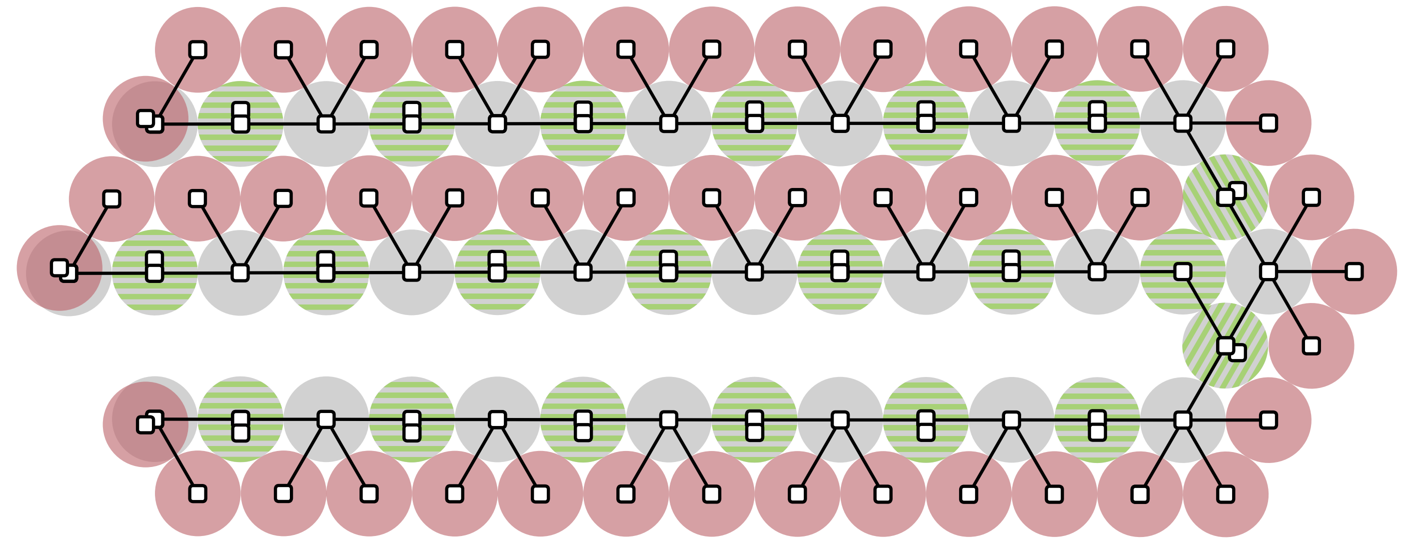


λ-Stable Hexagons Approximation

$\mathcal{O}(1)$-stable approximation of
a long thin rhombus



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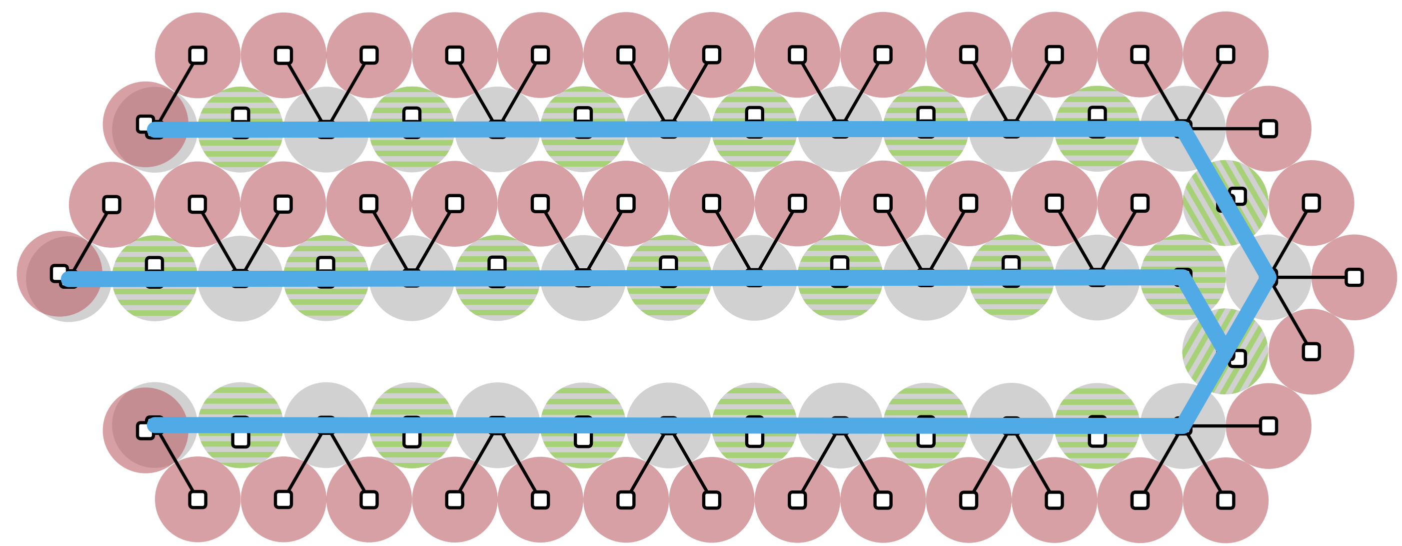

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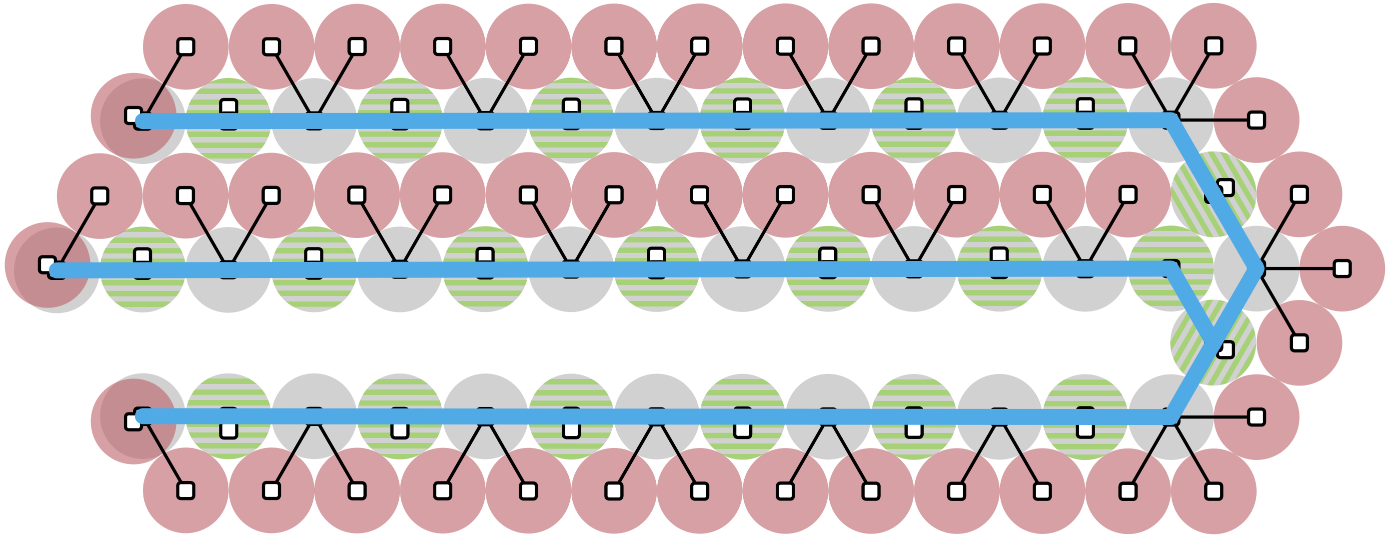

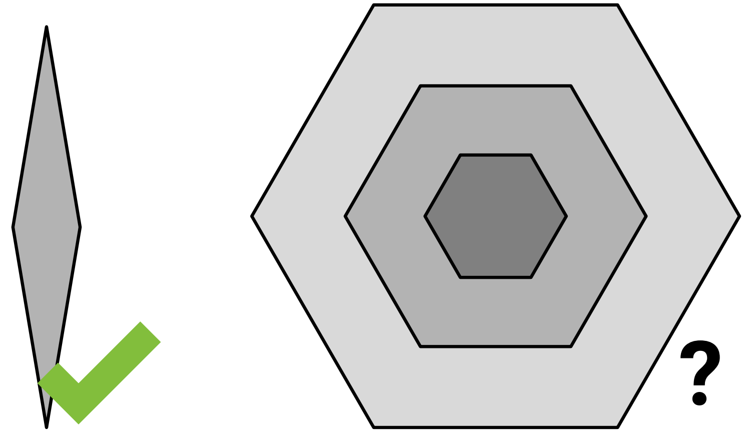

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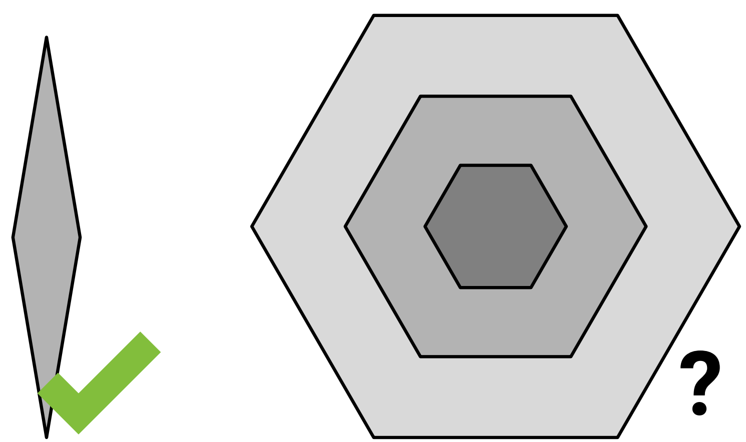

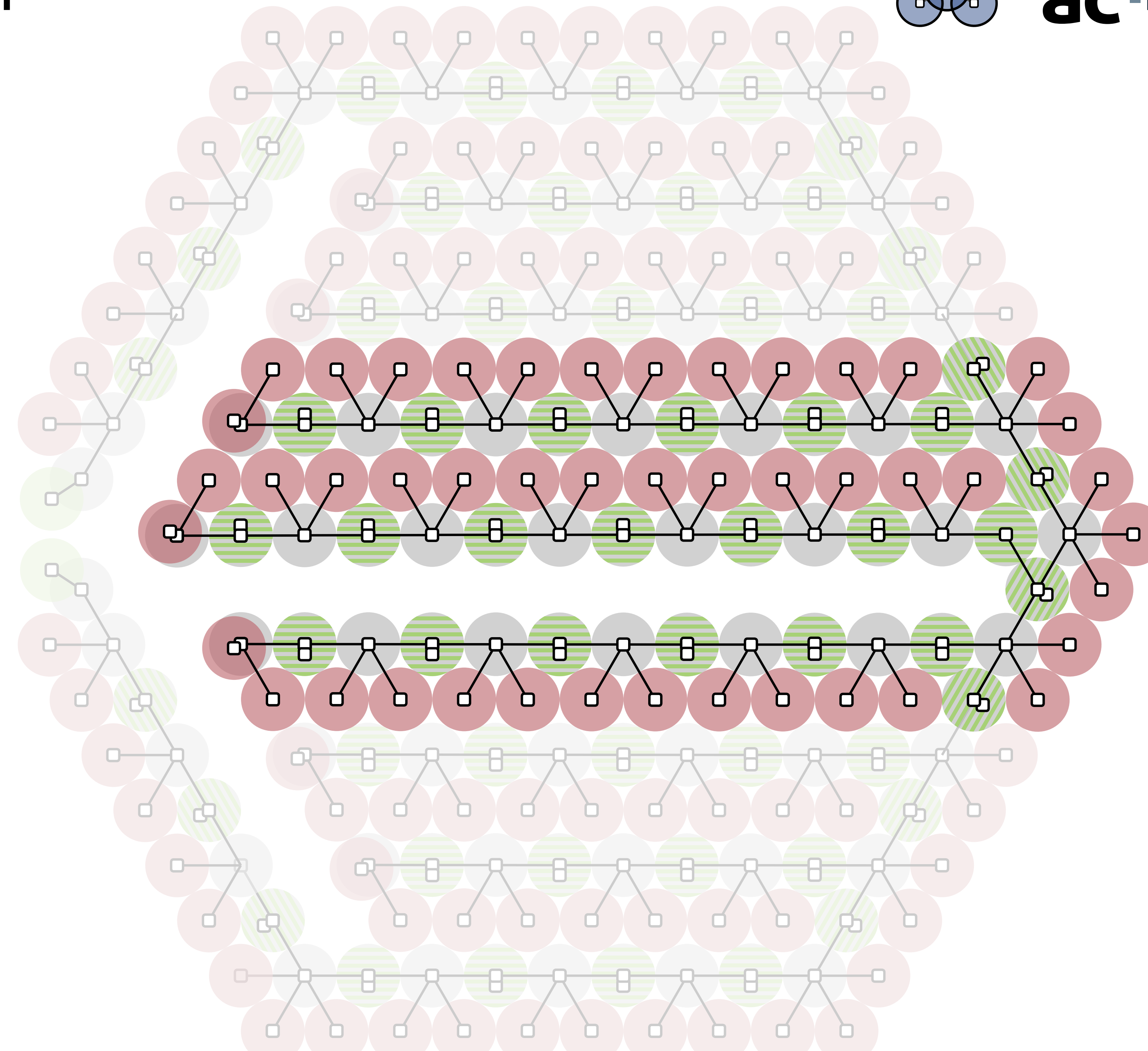

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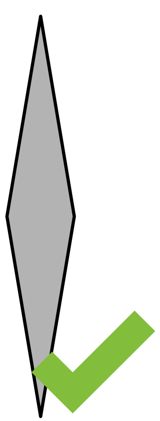

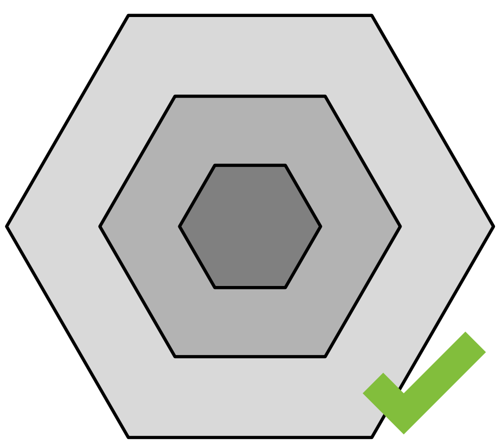

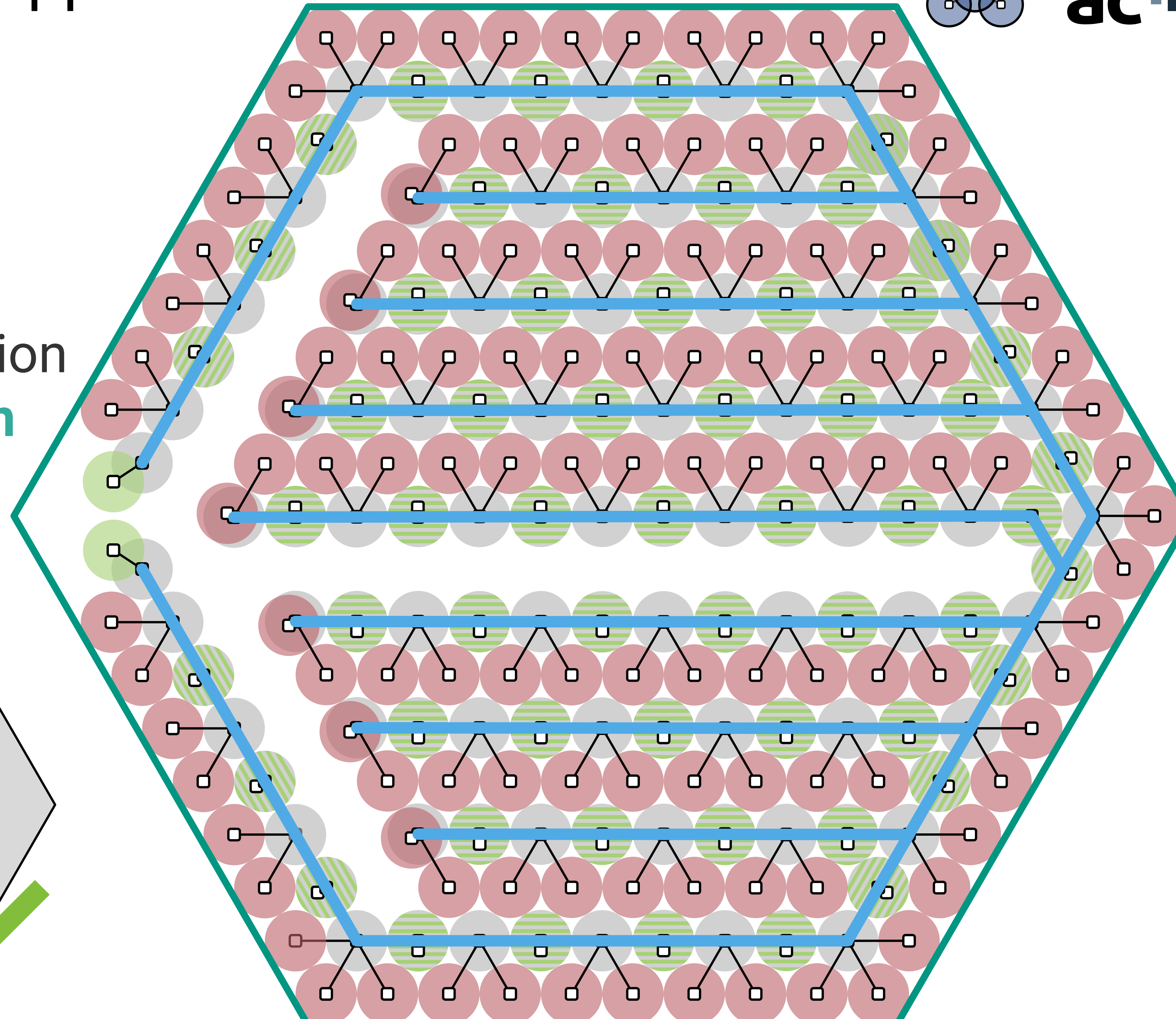

Hardness Results and Outerplanarity

Theorem 1. *Recognizing unit disk graphs is* NP*-hard for embedded trees.*

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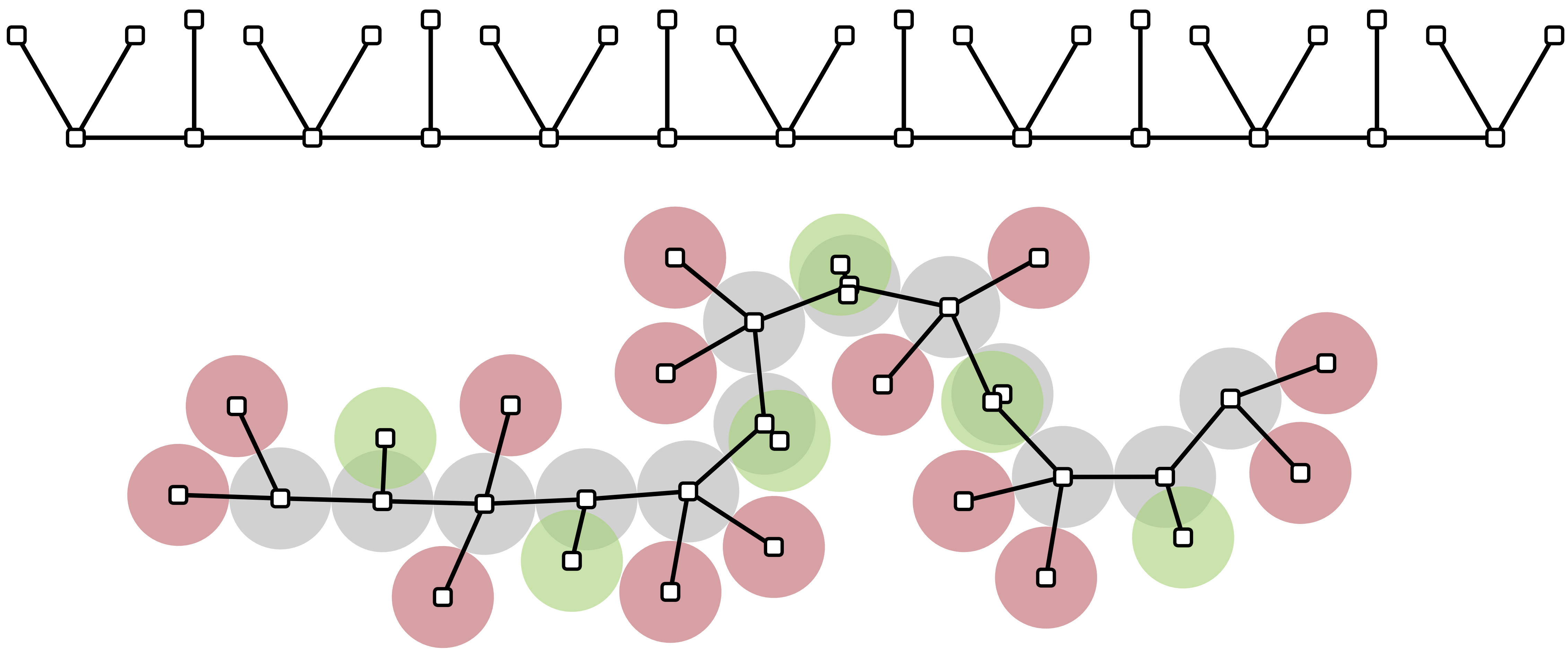

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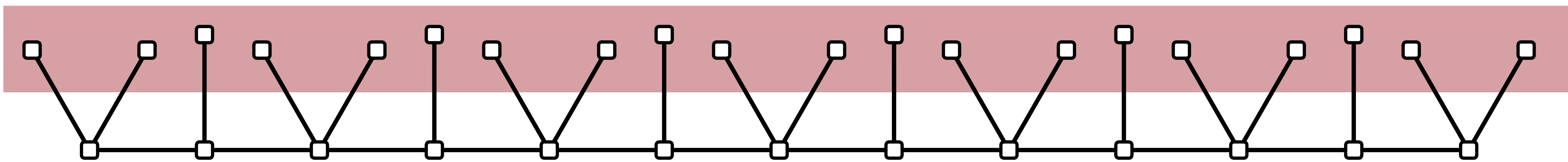
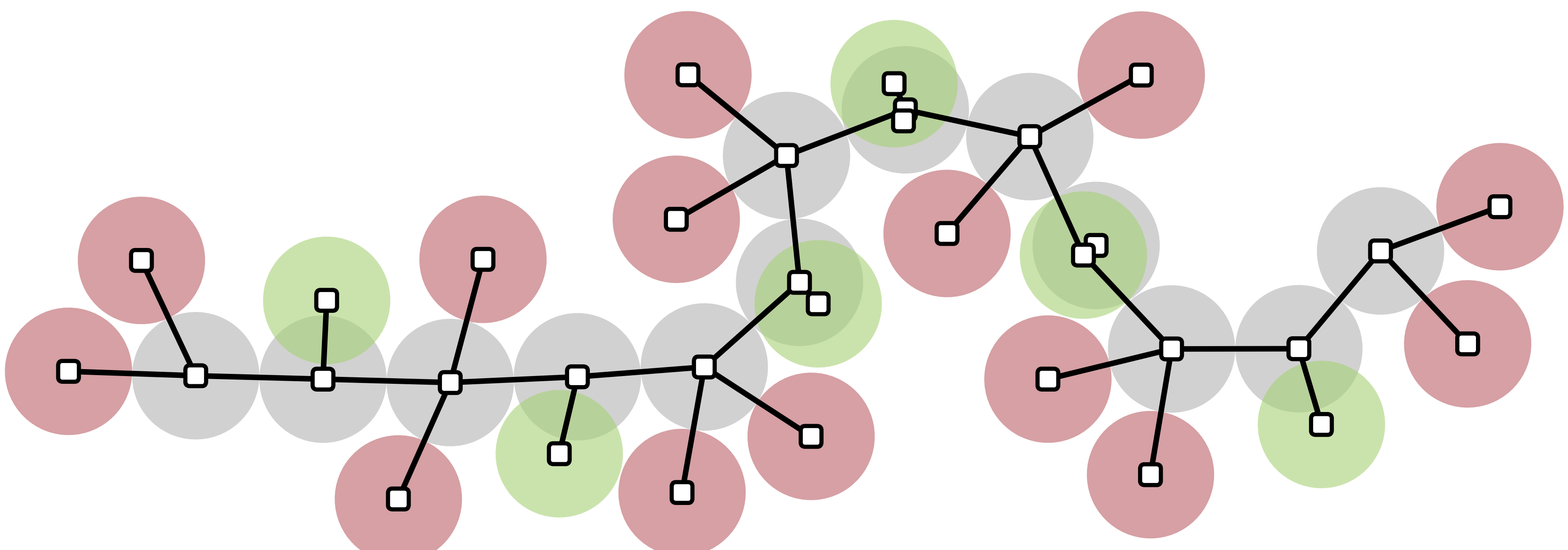
These need to be forced on one side





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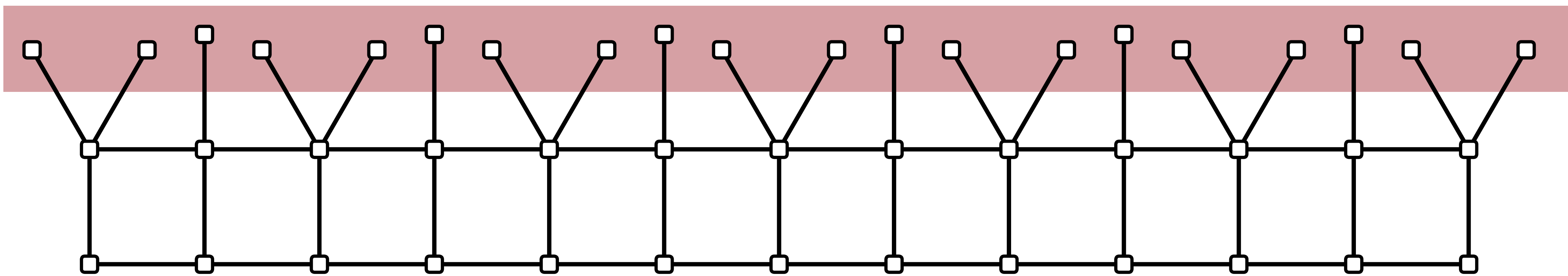

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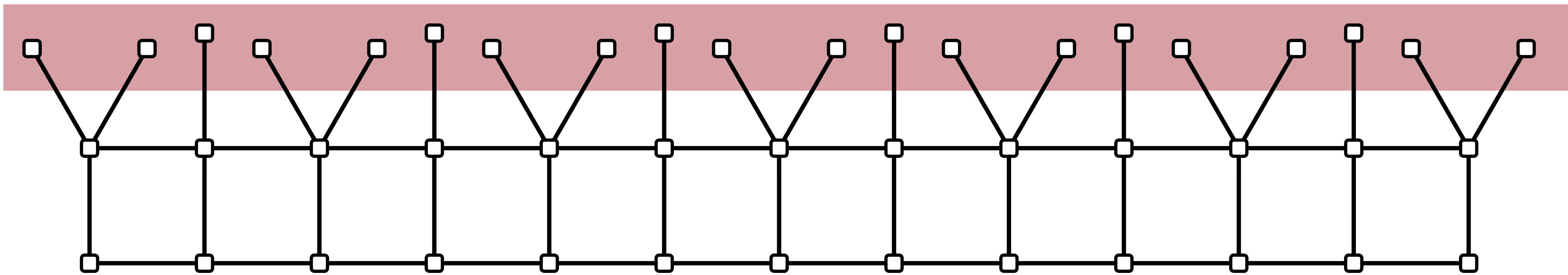
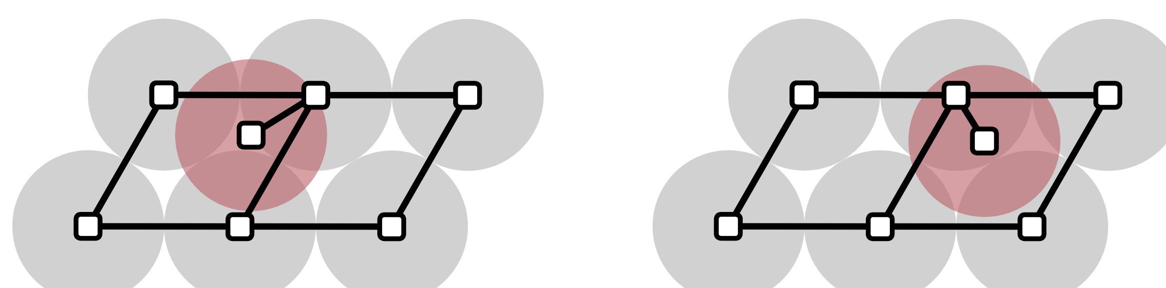


Outer neighbours can not be placed on the inside.



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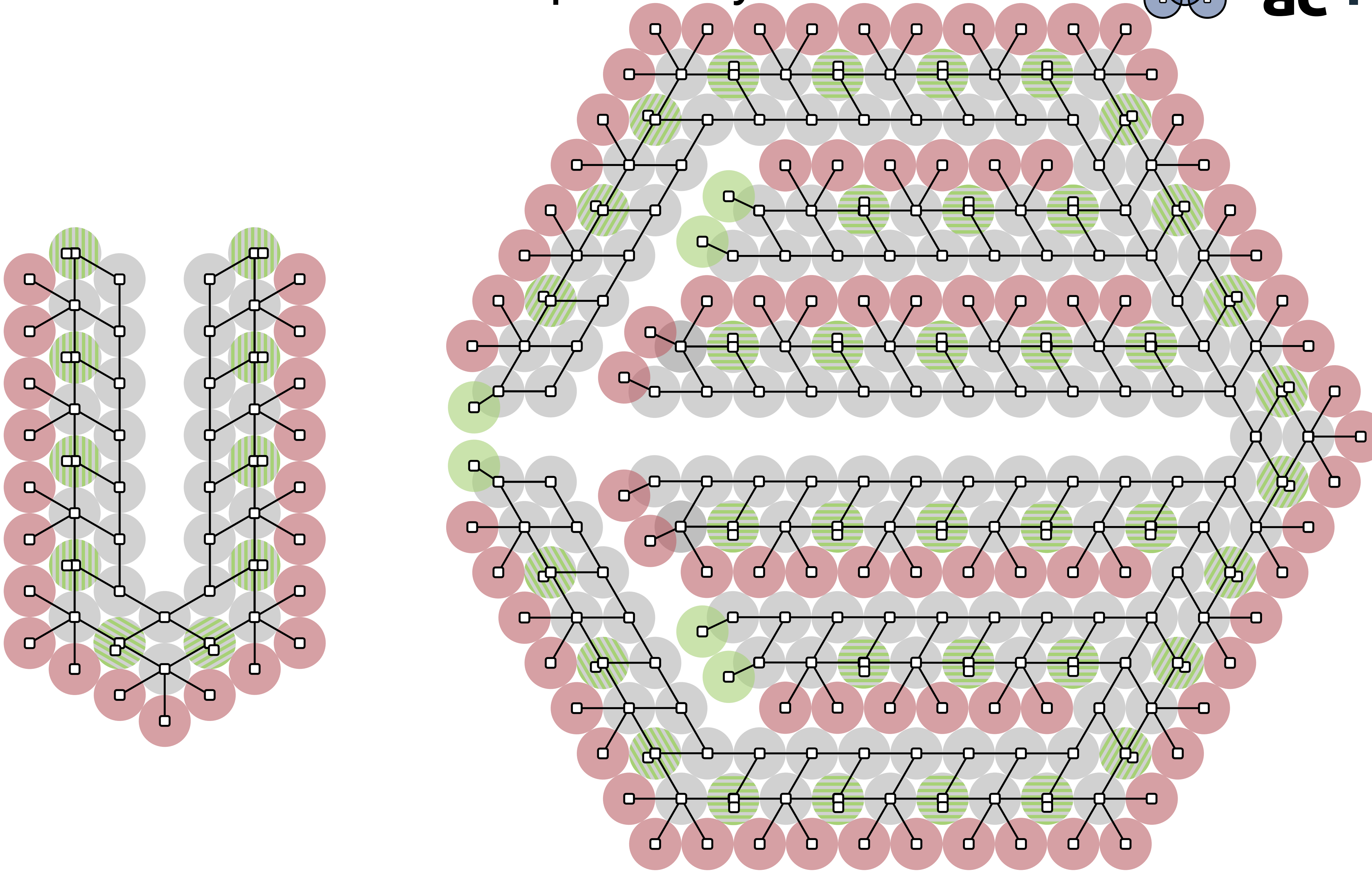

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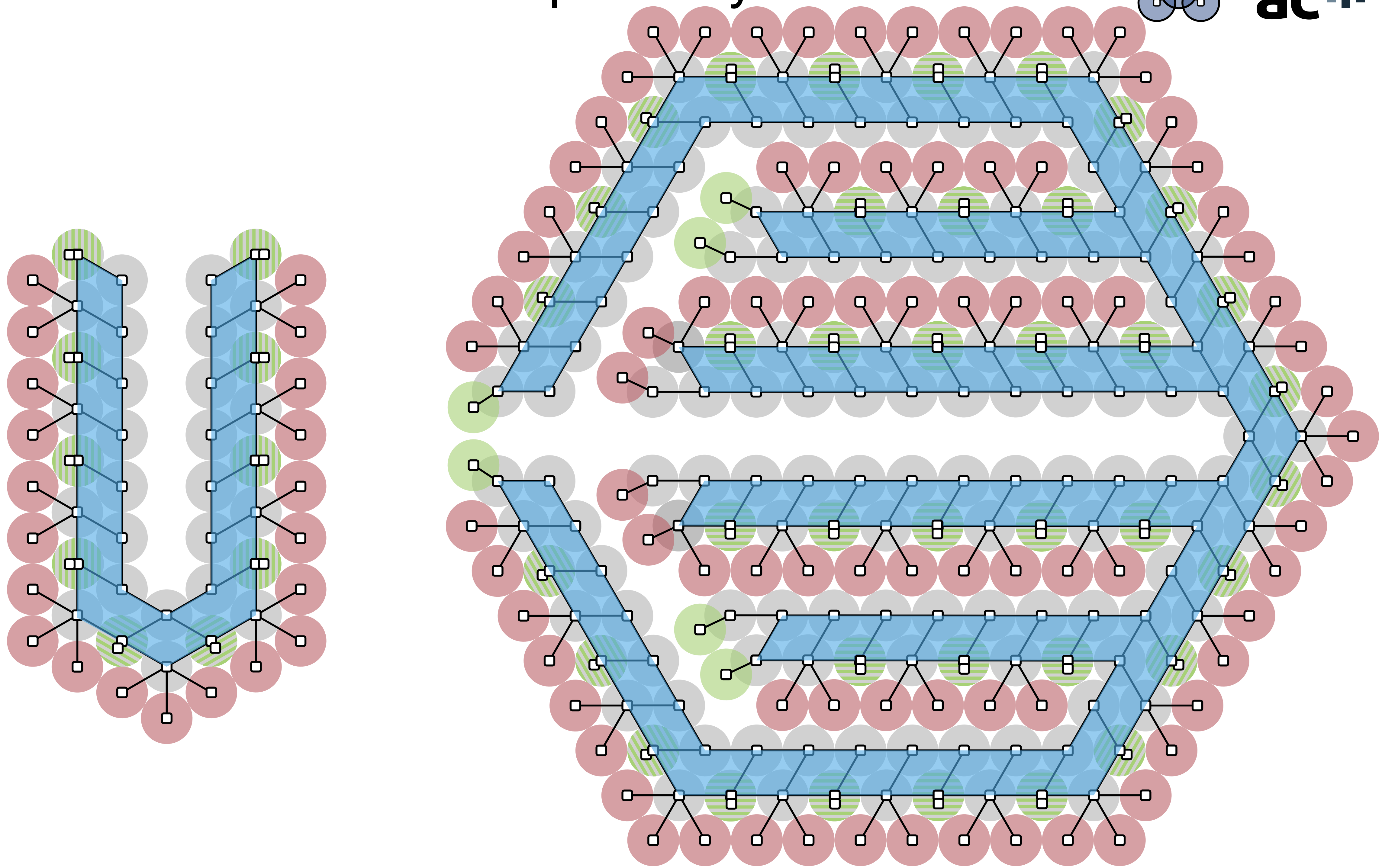

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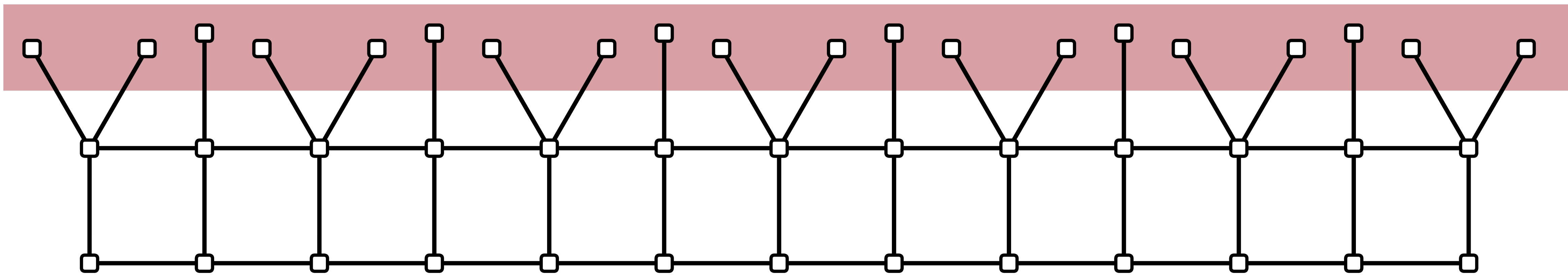
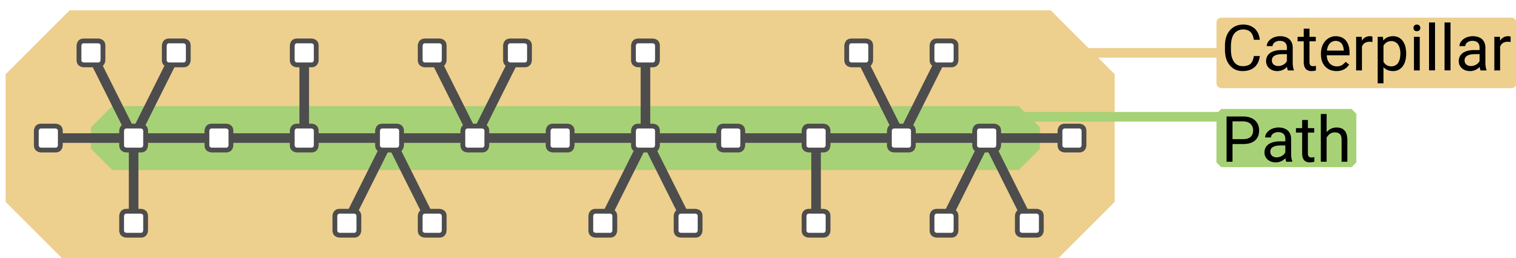


Theorem 2. *Recognizing unit disk graphs is* NP*-hard for non-embedded outerplanar graphs.*

Caterpillars admitting a UDR - Characterization

Caterpillars without embedding

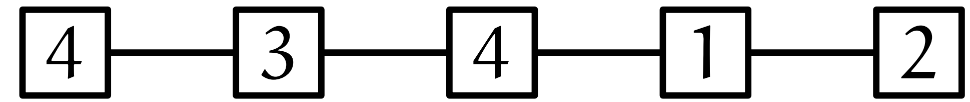


Caterpillar ≙ Path with leaves

Caterpillars admitting a UDR - Characterization

Caterpillars without embedding

A caterpillar C



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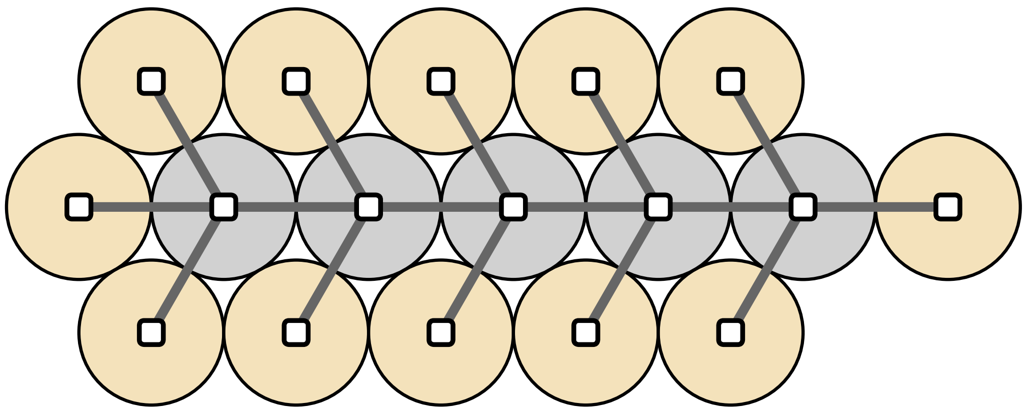
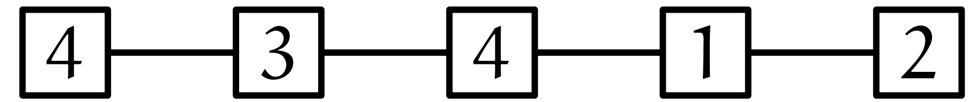


4-regular spines

A caterpillar $\mathcal{C}$



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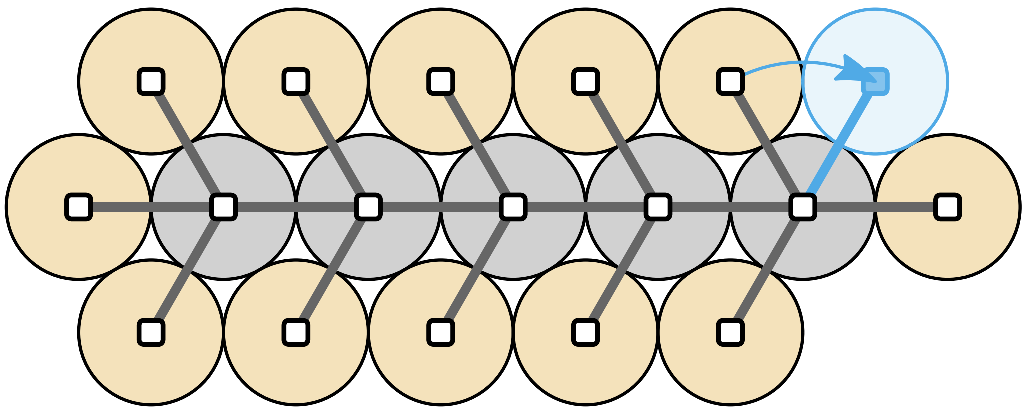

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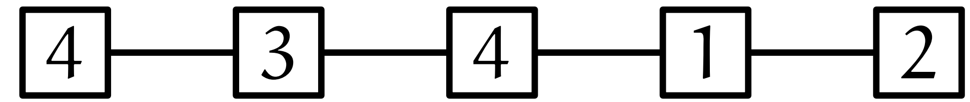
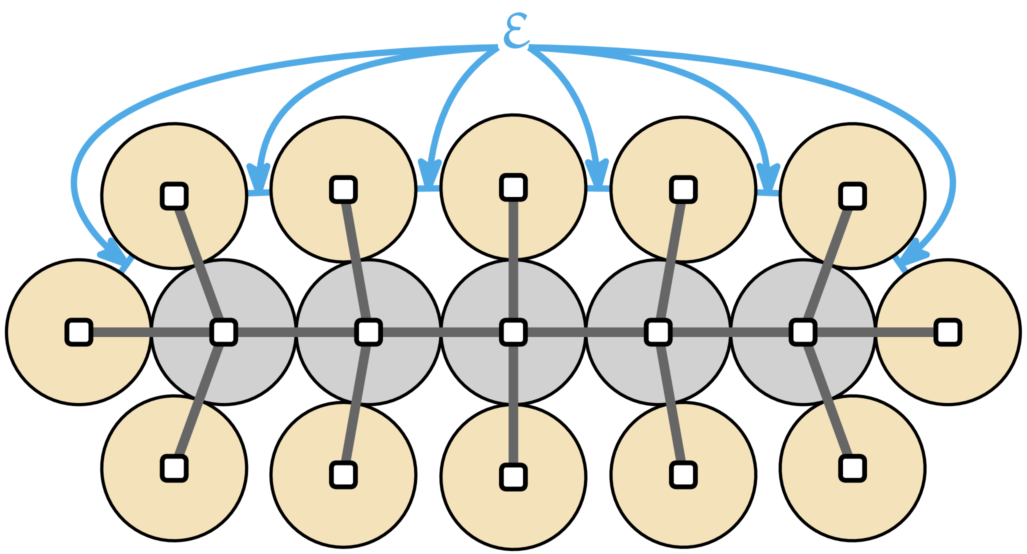
A caterpillar C



Caterpillars admitting a UDR - Characterization

Caterpillars without embedding



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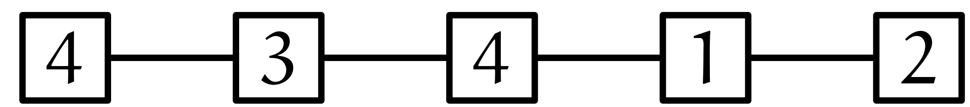

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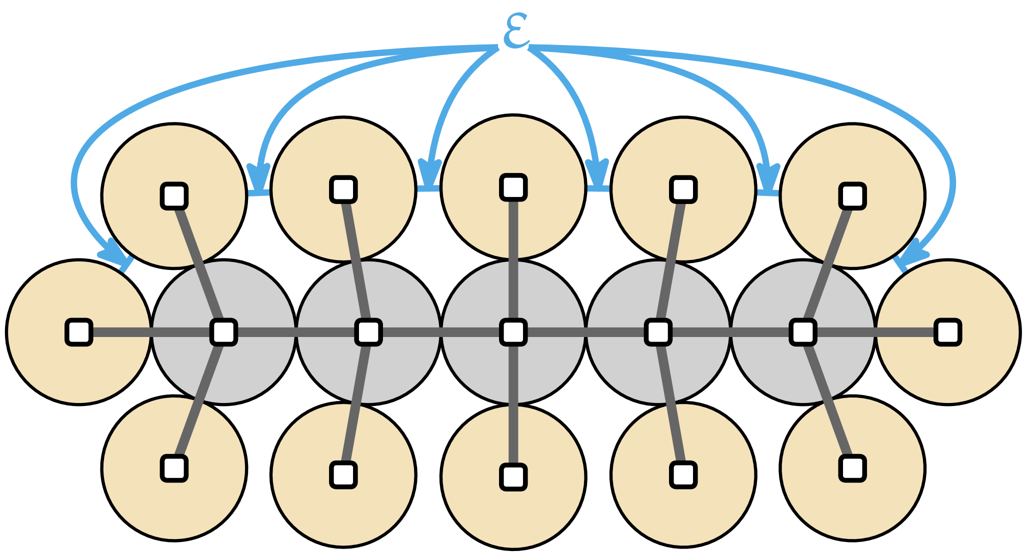

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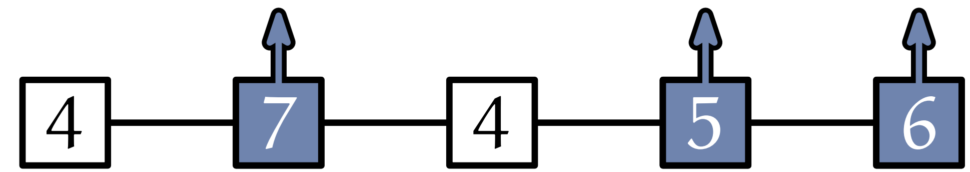

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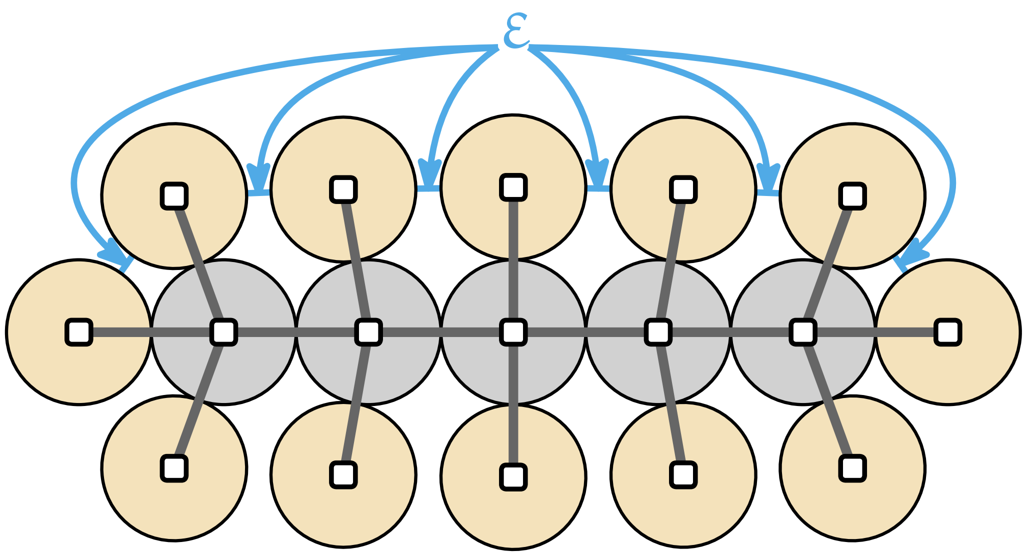

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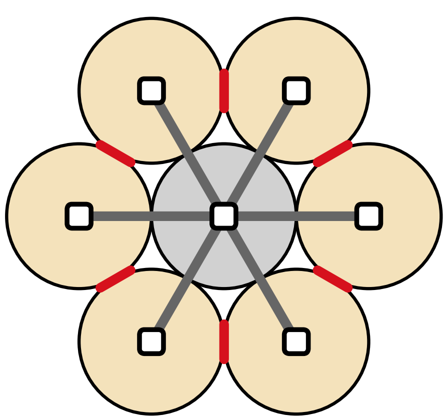

Degree 6 vertices are **impossible**!

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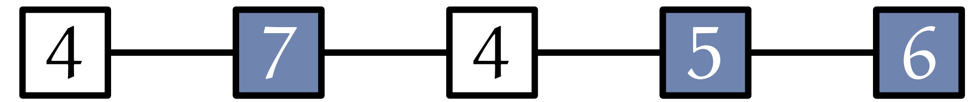

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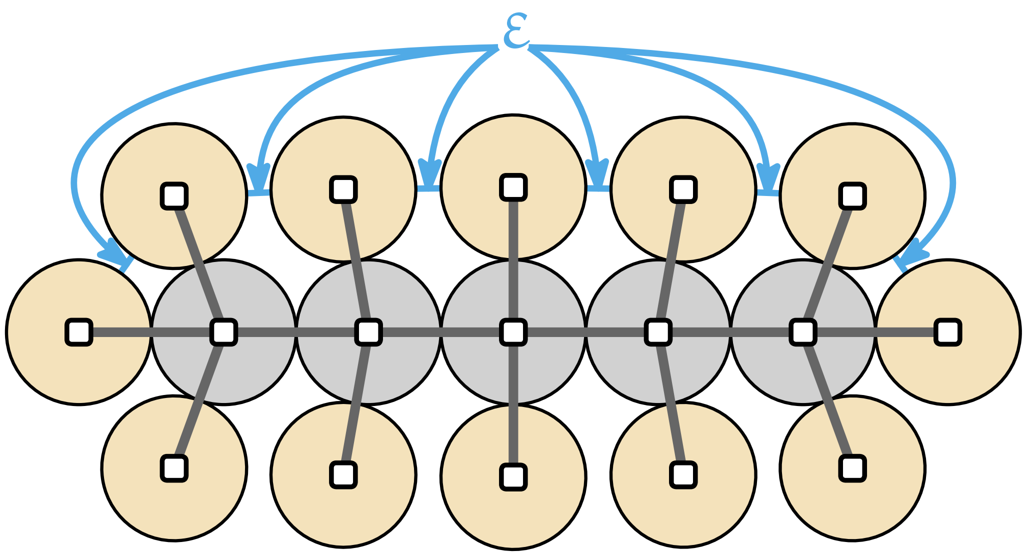

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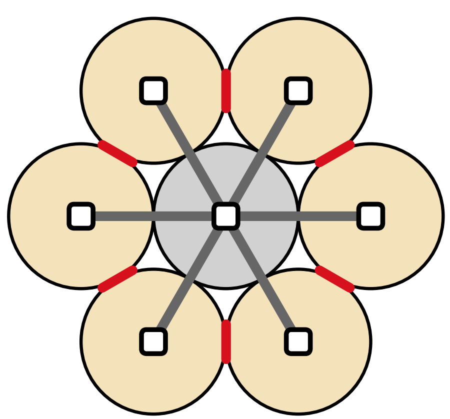

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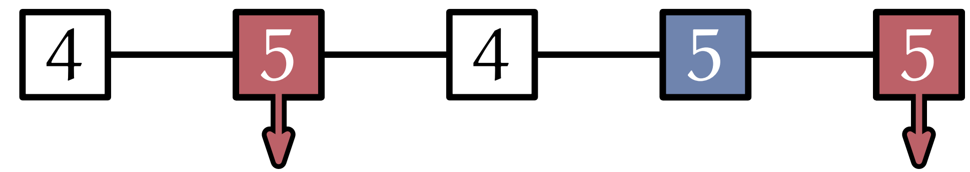

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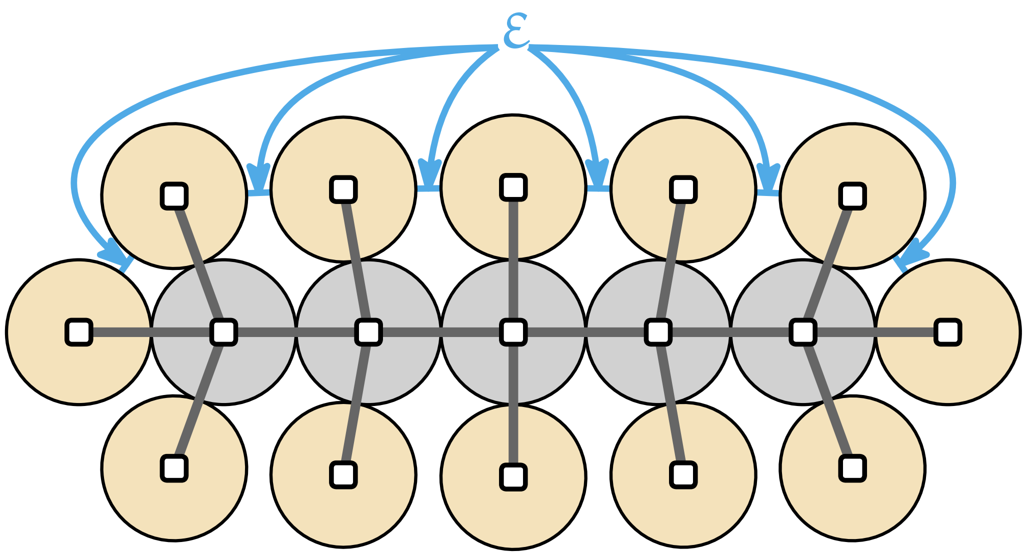

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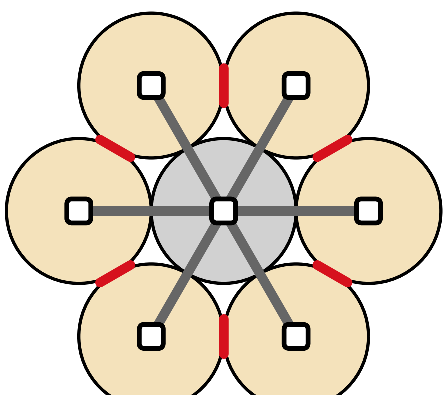

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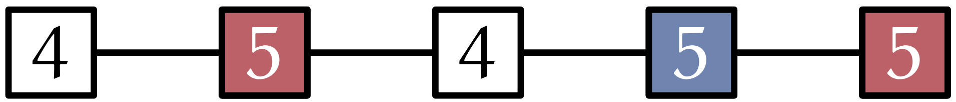

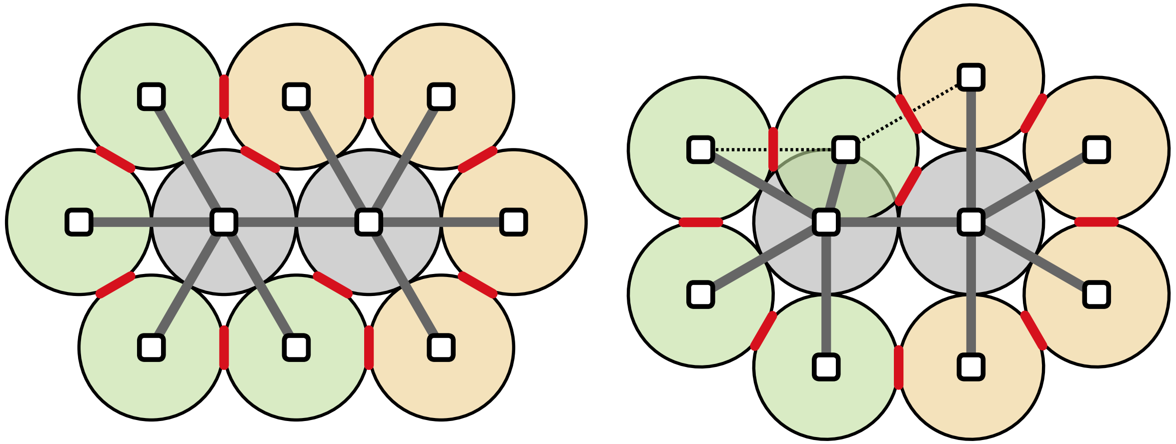

Consecutive degree 5 vertices are impossible!

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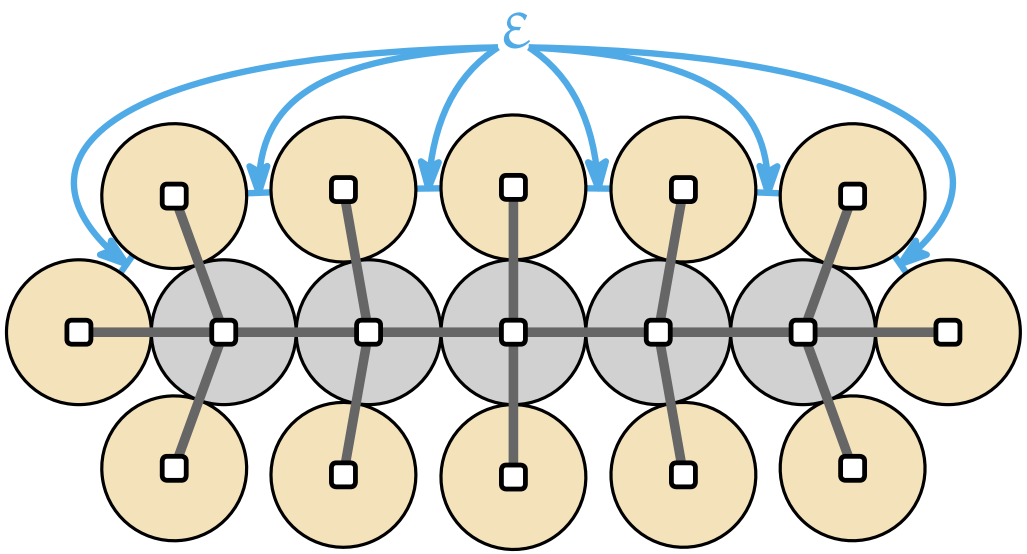
Caterpillars without embedding



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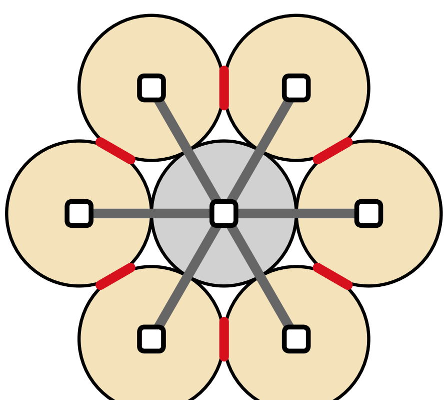

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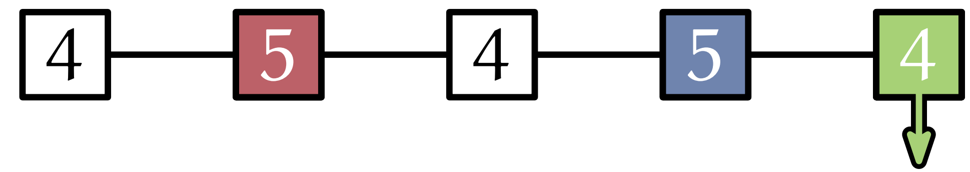

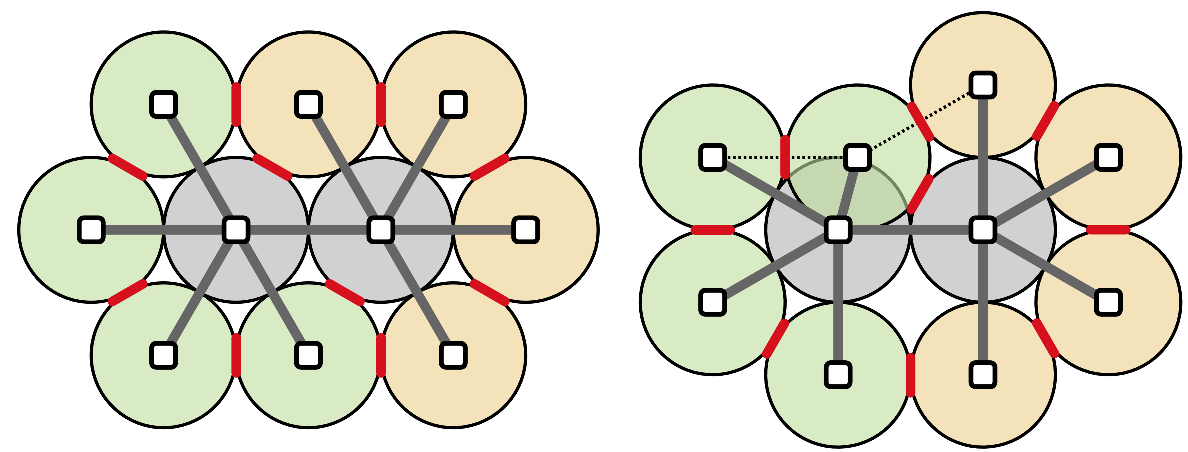

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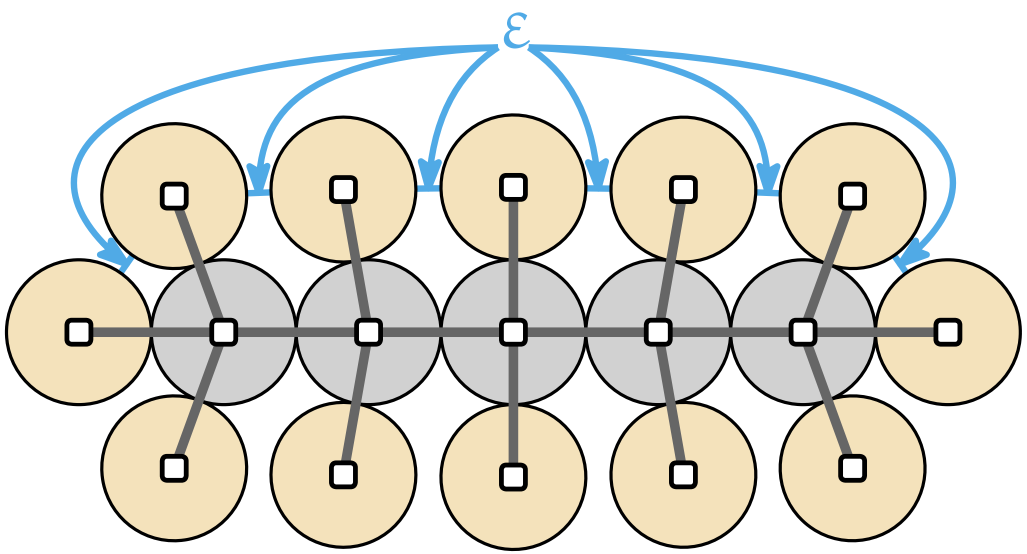

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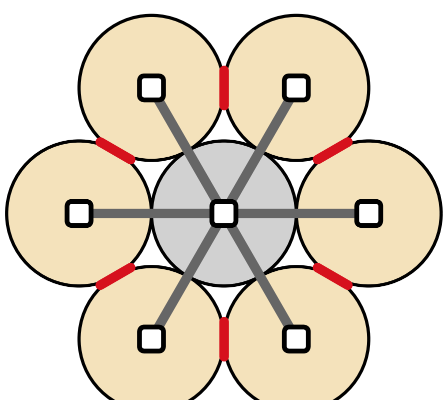

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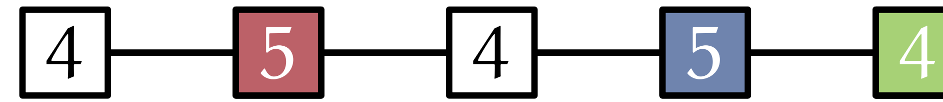
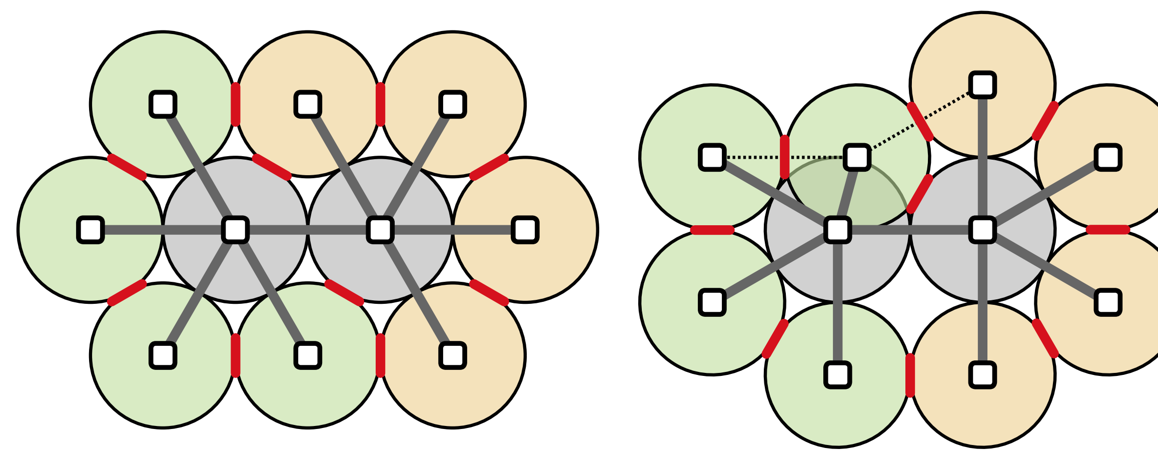


Can we realize $C = \ldots 5454545 \ldots$?



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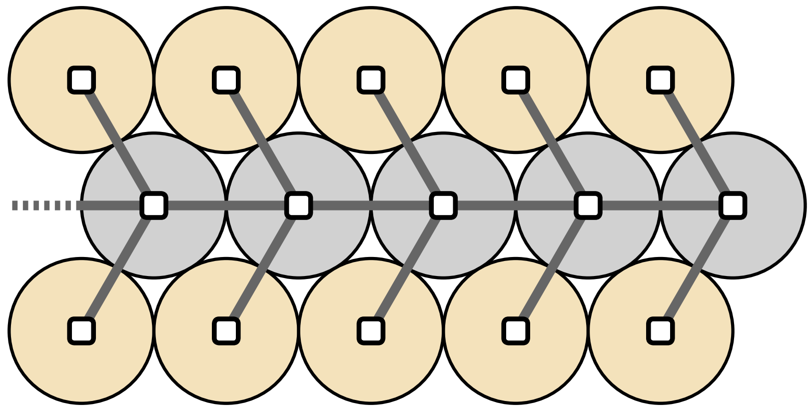
Caterpillars admitting a UDR - Characterization

Caterpillars without embedding

Backward position,
as long as degree is ≤ 4



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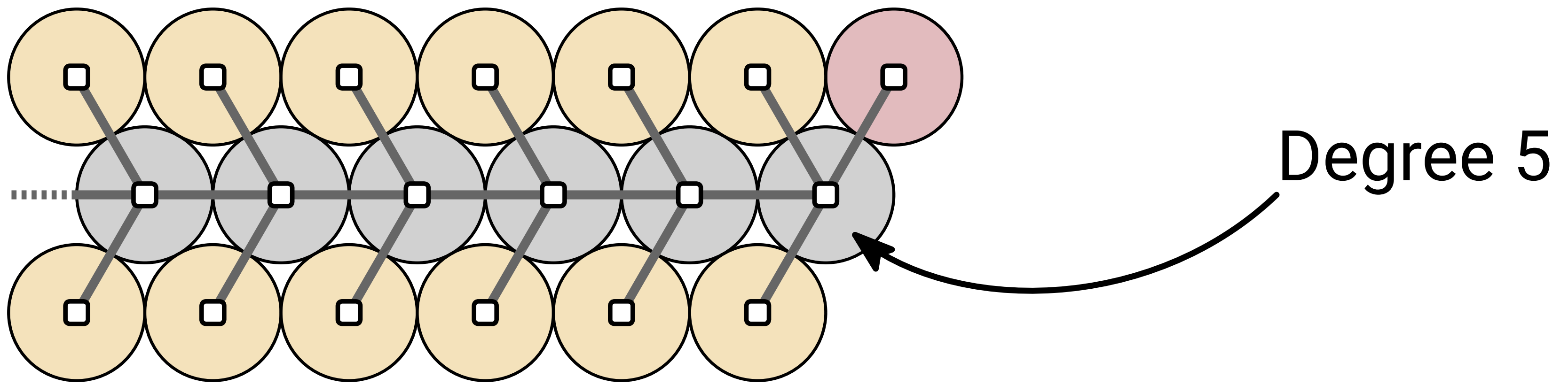

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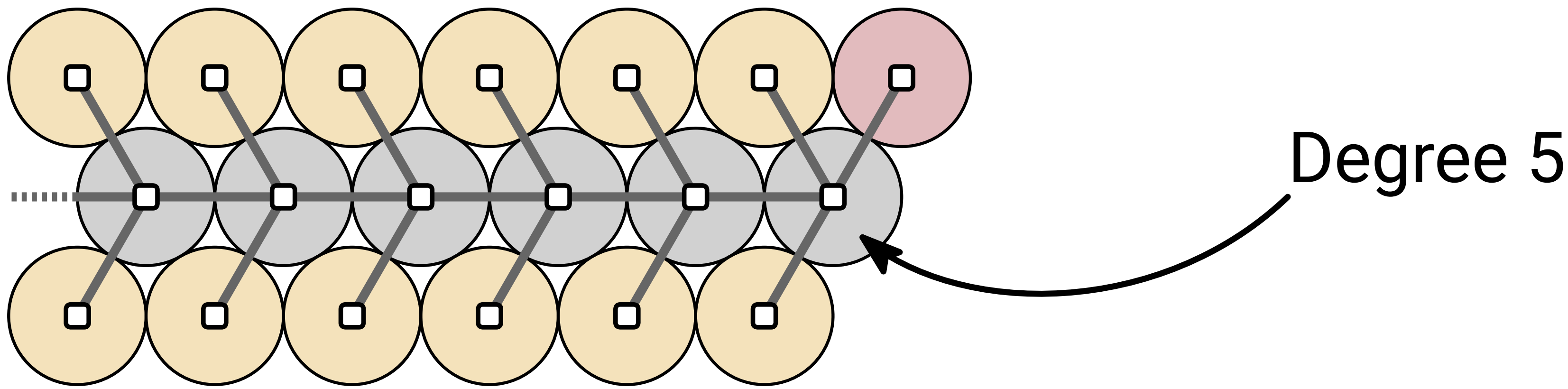

- If the next vertex has degree 5, we abort.

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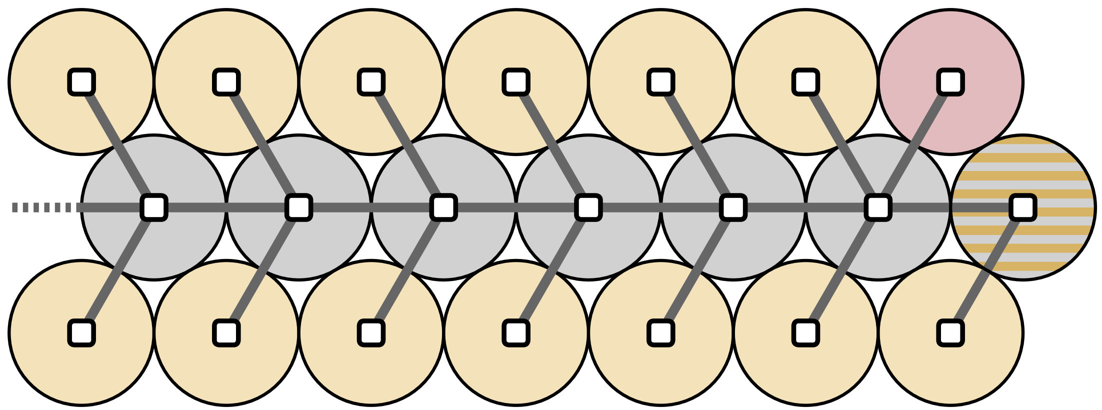

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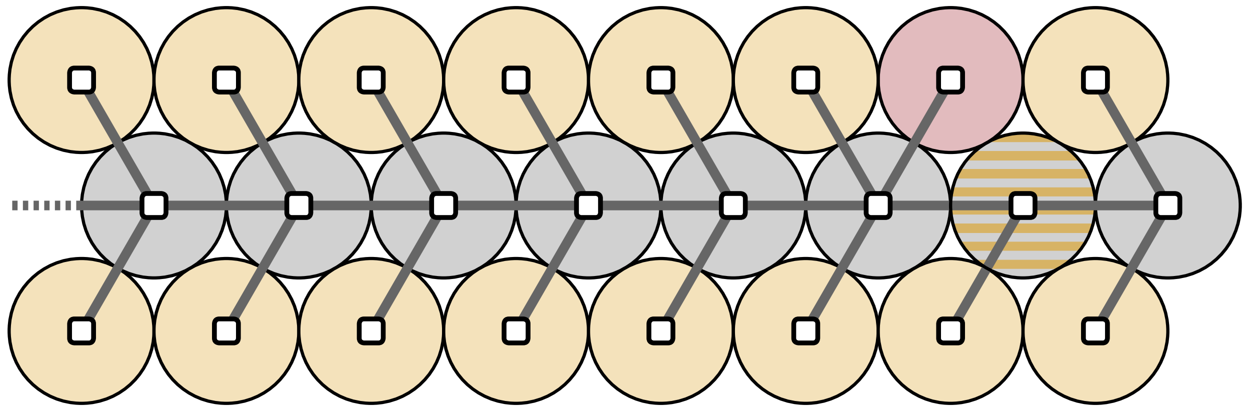

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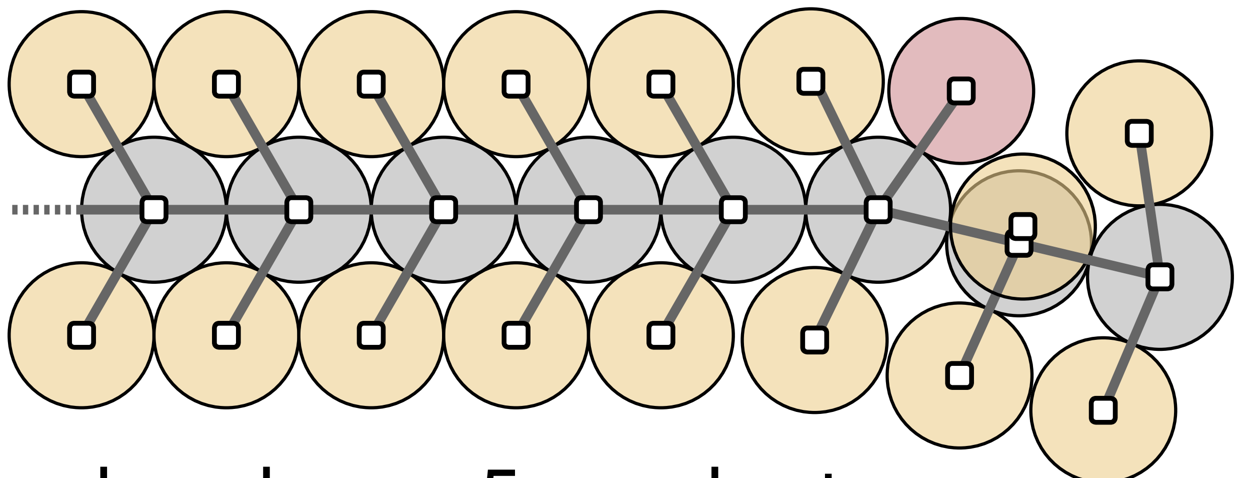

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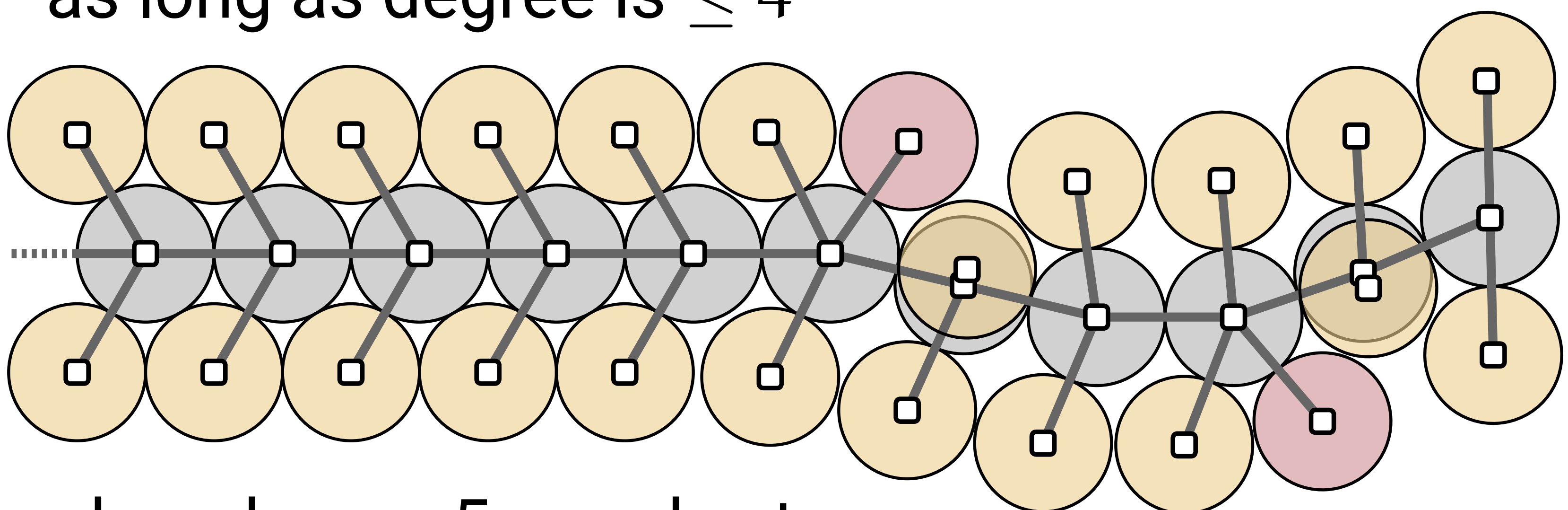

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- At the end, use available slack again to avoid wrong adjacencies

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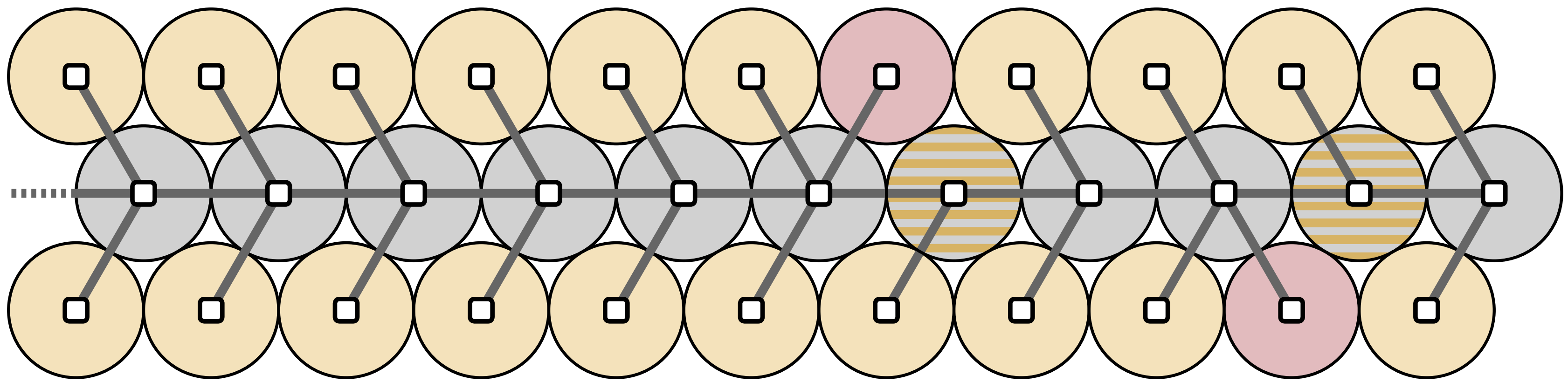

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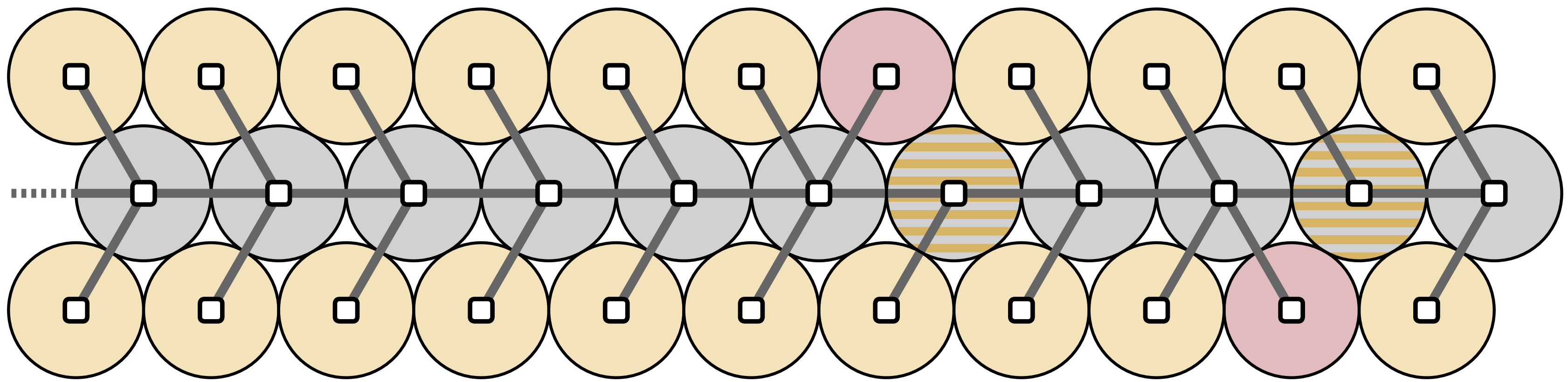

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Theorem 3. *Caterpillar graphs admit a UDR, if and only if they neither contain vertices of degree 6 or higher nor two adjacent vertices of degree 5.*

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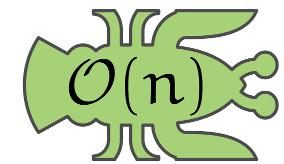

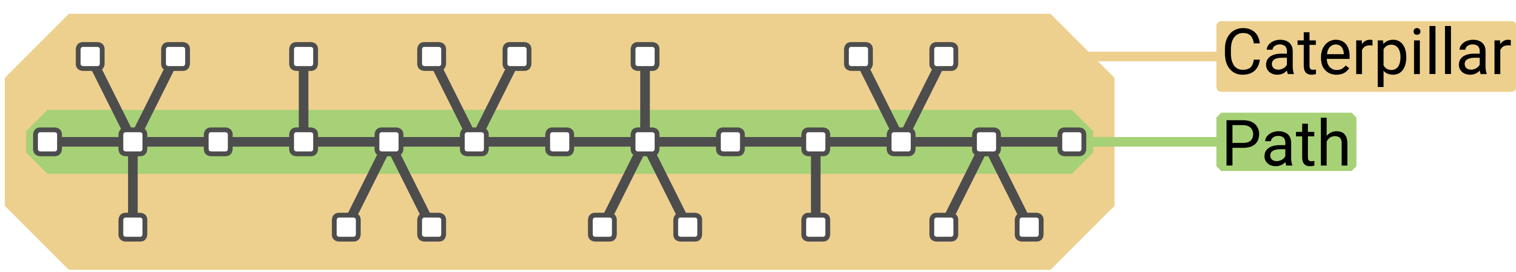

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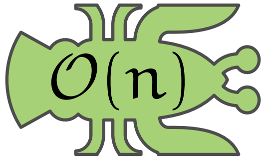

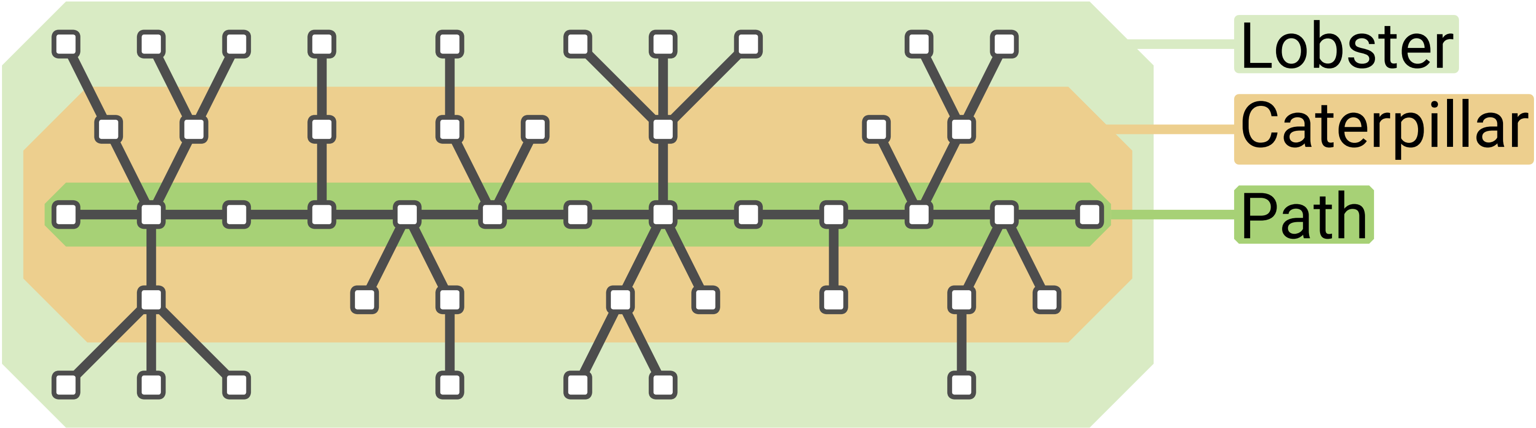

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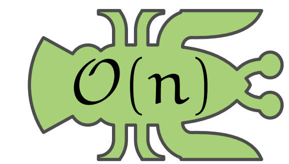

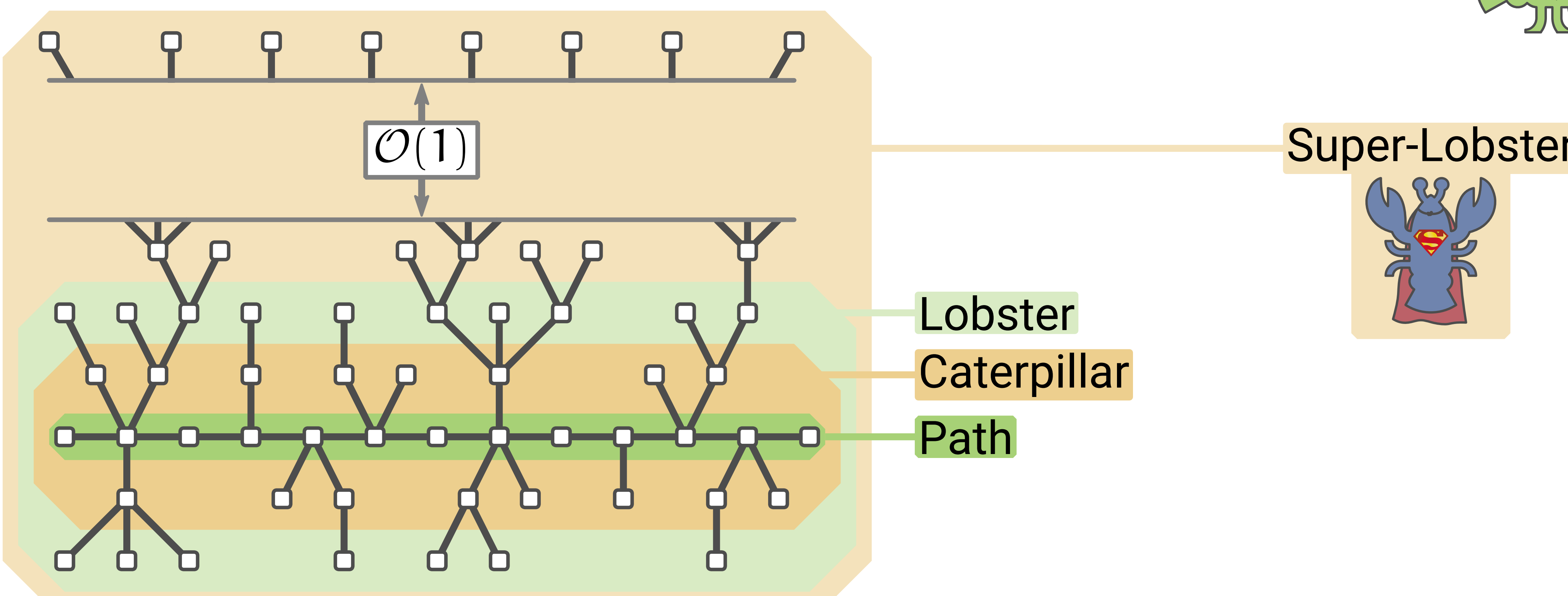

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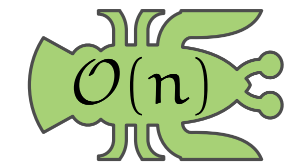

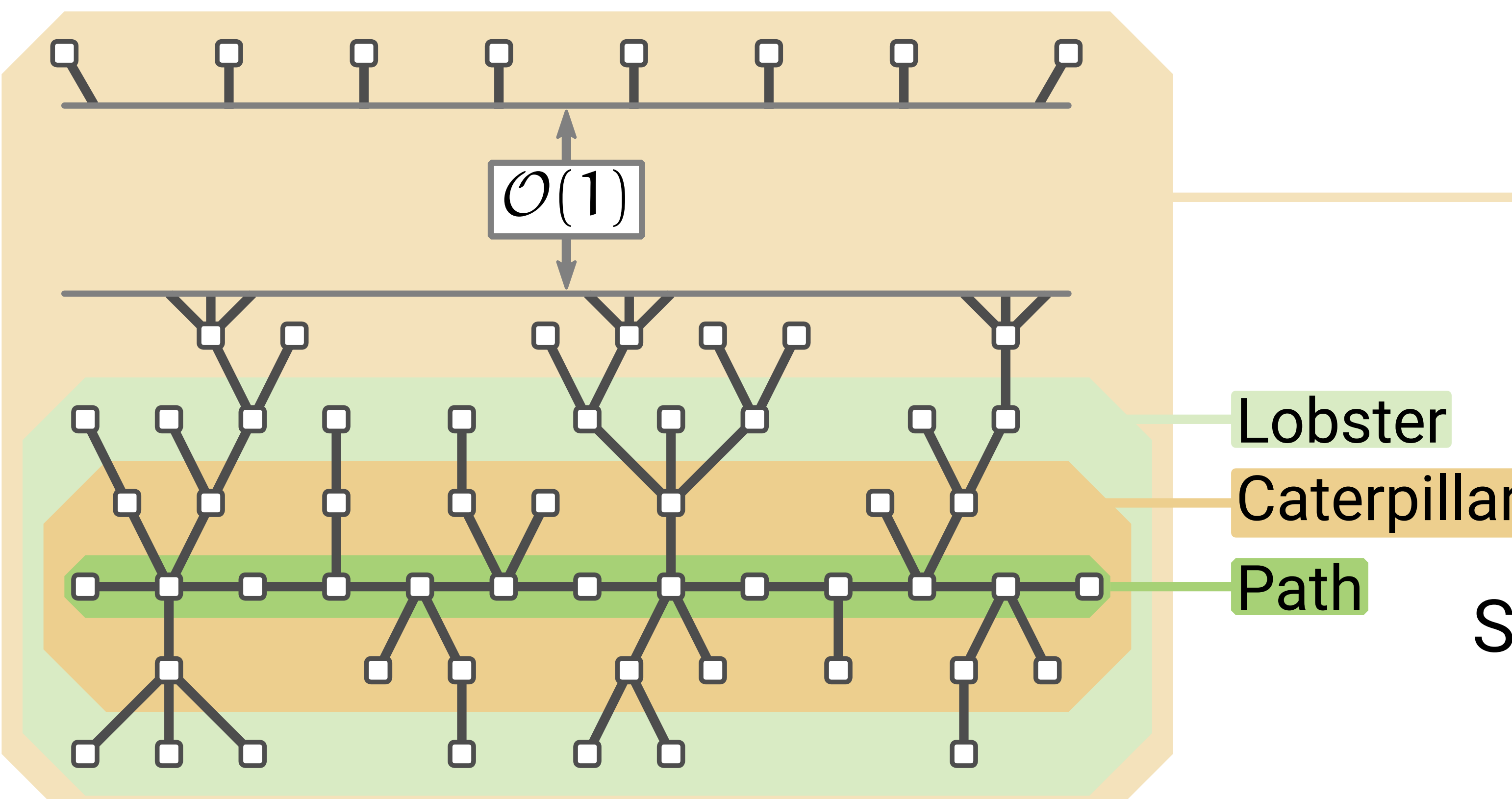

Weak UDCs for Super-Lobster in $\mathcal{O}(n)$ time?

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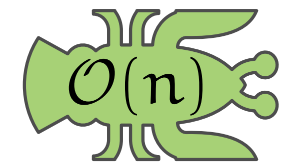

Open questions:

	tree	caterpillar
contact graphs	**?**	$\mathcal{O}(n)$
↳ embedded	NP-hard	**?**
disk graphs	**?**	$\mathcal{O}(n)$
↳ embedded	NP-hard	**?**

Super-Lobster

Weak UDCs for Super-Lobster in $\mathcal{O}(n)$ time?